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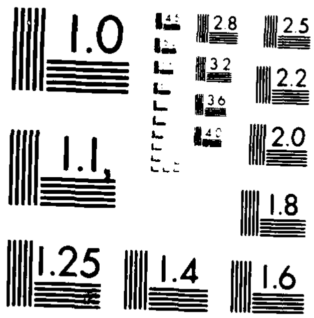

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NOVEL TRANSPORT AND RECOMBINATION PROCESSES IN SEMICONDUCTORS

ANNUAL TECHNICAL REPORT

by

M. PEPPER

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ABSTRACT

This report contains descriptions of our work on two dimensional transport in Si and InP devices and spin dependent recombination in Si gate controlled p-n junctions. The two dimensional work covers our investigations of the quantum Hall effect where we find that the effect is basically d.c. and can be measured by the application of a finite frequency. The cause of this effect is discussed in terms of a delocalisation of electrons in the tails of Landau levels. The experiments indicate that localisation in the tails of Landau levels is caused by both disorder and the electron-electron interaction. Another aspect of the electron-electron interaction which has been investigated is the oscillatory conductance in inversion layers when charge is present at the Si-SiO_2 interface. It is suggested that Coulomb effects give rise to a contribution to the activation energy which oscillates with carrier concentration. Other topics in two dimensions which are discussed include the role of spin-orbit coupling in transport in the InP inversion layer and an investigation of the scaling theory of conduction. In this latter topic it is concluded that a one parameter scaling function does not exist. The existence of the weak 2D localisation has also allowed an investigation of the rate of energy loss of hot electrons in Si inversion layers.

The other topic discussed is spin dependent recombination in Si gate controlled p-n junctions. The signal is found to be independent of frequency as suggested by theoretical models. We have also found a spin dependent generation signal of the same magnitude as the recombination signal. At present we do not have a theoretical model for this effect, future work will include both experiments and an attempt to produce a model accounting for both spin dependent recombination and generation.

A-1

1. INTRODUCTION

In this, first, report we present results of our investigations into various aspects of two dimensional transport and spin dependent recombination. The two dimensional work has been published and the papers are attached as an appendix, thus the main part of this report will only contain short summaries of the papers. A fuller description of the work on spin dependent recombination is given. Work on the ballistic injection of electrons is now at the stage of producing results but there is nothing suitable, at present, for incorporation in the report.

2. TWO DIMENSIONAL TRANSPORT

(a) The Wigner glass and conductance oscillations in silicon inversion layers.

We show that on changing the nature of the background random potential the inversion layer of the Si MOSFET exhibits the conductance oscillations previously observed in both GaAs, the source and drain regions of Si MOSFETs and MOSFETs with a very high concentration of Na^+ ions at the Si-SiO_2 interface. Measurements of the temperature dependence of conductance show that oscillations are found when conductance is by an excitation process as well as hopping. The oscillations arise from an oscillating activation energy which is due to either an oscillating interaction contribution to the activation energy, or a negative effective density of states at certain values of carrier concentration. This appears due to electron ordering in a small current limiting region and is contrasted with the case where the oscillations are absent, although localisation is due to both background disorder and the random field of localised electrons. It is shown that near the transition, localisation is due almost entirely to the random field of the localised electrons, and the system is a strongly interacting Fermi glass (or Wigner glass), even though the oscillations are not apparent.

(b) Electron localisation and the 2D quantised Hall resistance

Using Si inversion layers we have investigated the plateau of quantised Hall resistance appearing when the Fermi energy E_F is between the spin parallel and spin anti-parallel states of the ground Landau level. When states below E_F are localised, indicated by a temperature-dependent σ_{xx} throughout the level, a plateau is not formed; subsequent delocalisation of states near the centre of the level results in the appearance of the plateau. The delocalisation can be achieved by an increase in temperature, or the application of a substrate bias, and the ease of this process indicates that the degree of localisation, when present, is weak. Under these circumstances the localisation is long range and can be interpreted as the absence of a continuous extended path through the specimen.

Measurements under AC conditions result in the appearance of a plateau with increasing frequency when one is not apparent at DC. This is discussed in terms of the localisation length, and it is suggested that electrons behave as if delocalised when the frequency is such that the drift length is less than the localisation length.

The behaviour of ρ_{xy} throughout the ground Landau level has been analysed and it is shown that a 'normal' Hall effect is not obtained from extended states at the centre of the level. This is in agreement with theories which suggest that the extended state σ_{xy} compensates for the presence of localised states in the tail of the level. It is found that when extended states are present in the second level (0↑ −) they not only compensate for the localised states in this level but also compensate for the bottom level (0↑ +) which is entirely localised. On the other hand, when states near the centre of the level are localised, and a plateau is not found, the results indicate that weakly localised carriers contribute normally to the Hall effect. The absence of any effect of the weak localisation on σ_{xy} is the same as has been found for weak, non-magnetic localisation in both two and three dimensions.

(c) Spin-orbit coupling and weak localisation in the 2D inversion layer of Indium Phosphide

We report measurements of the magnetoresistance (MR) of the 2D inversion layer of an InP MOSFET in the temperature range $T = 4.2 \rightarrow 0.3K$. For $k_F l > 5$ we observe positive MR at low magnetic fields $B < 0.015$ T and negative MR at higher values of B. We attribute this behaviour to the presence of strong spin-orbit coupling which at B = 0 reduces the magnitude of the weak localisation. As B is increased, the spin-orbit interaction is progressively quenched, leading to a positive MR which eventually turns over into negative MR as the time reversal symmetry of the quantum interference (weak localisation) is destroyed. Our results can be qualitatively explained but precise agreement could not be obtained in the regime where the spin-orbit effect was dominant.

Analysis of the negative magnetoresistance has allowed us to observe the role of electron-electron scattering which can be expressed as two terms varying with temperatures as T and T^2. The results do not support a suggestion that the T term can be expressed as $T\ln(T_1/T)$ where T_1 is a constant.

(d) An experimental test of the scaling theory of conduction in two dimensions

The scaling theory of conduction has created much interest in the problem of conduction in a disordered system. Basic to this theory is the assumption that a one-parameter scaling function exists. An experimental test of this is presented for two-dimensional transport in silicon inversion layers. The results are found to be inconsistent with such an assumption and we conclude that the function does not exist.

(e) Energy loss rate in silicon inversion layers

We report the results of measurements on the rate of heat loss from hot electrons in silicon inversion layers at low temperatures. The results are interpreted in terms of the generation of acoustic phonons and it is found that disorder has a significant effect on this mechanism. In the low-disorder, high-temperature limit the energy relaxation time τ_ε varies with electron temeprature T_e as T_e^{-4}. In the high-disorder, low-temperature limit τ_ε varies as T_e^{-2}. The electron temperature is measured by the effect on the weak two-dimensional localisation which allows the experiment to be performed at low temperatures.

(f) Loss of dimensionality, localisation and conductance oscillations in n-type GaAs FETs

We present new results for the transition from 3 dimensional (3D) conduction to 2D conduction in a GaAs FET. By applying a magnetic field, B, it is possible to observe 2 metal-insulator transitions at low temperatures by (a) suppression of weak localisation at low B returning the system to metallic conduction and (b) shrinking of the donor wavefunctions at high B localising states at the Fermi energy. Magnetoresistance has been measured over 4 decades of B and for temperatures between 4.2 and 1.2 K, the results being in satisfactory agreement with current theories of localisation in 2D and 3D. We also present new conclusions for the anomalous oscillations in conductance with applied gate bias observed in most GaAs FETs at low temperatures. The strength of the oscillations is related to the quality of the FET, being predominant in commercial microwave GaAs FETs.

(g) Electron localisation and the quantized Hall resistance in silicon inversion layers

We have investigated the formation of the plateau of quantized Hall resistance in the spin split minimum and the lowest valley split minimum of the ground Landau level of (100) Si inversion layers. The results in the spin gap are explained by a model based on Anderson localisation in strong magnetic fields and on the existence of long range potential fluctuations. The behaviour of ρ_{xy} in the second, spin up, higher valley, level is discussed in relation to the compensation effect suggested in recent theories. Application of a field of 25 Tesla resulted in the delocalisation of electrons in the lowest valley level and the appearance of the plateau of quantized Hall resistance in the lowest valley gap.

3. SPIN DEPENDENT RECOMBINATION (SDR) IN IRRADIATED GATE CONTROLLED p^+n DIODES

The small bias forward current through a p^+n diode (Fig 1) is dominated by the recombination of electron-hole pairs in the depletion region. With larger bias (> +0.20V) the current is dominated more by the diffusion current contribution. In these experiments a circular gate controlled diode is placed in a magnetic field and the recombination current is enhanced by an amount ΔI when microwaves at 8.44 GHz are applied and resonance is established. The enhancement is measured as a first derivative of current, by phase-sensitive detection, at the frequency of modulation of the magnetic field. The magnetic field is swept slowly, and the spin-dependent enhancement occurs at about 3360G. The enhancement is a factor of up to 1 part in 10^3, and is of opposite phase to the first derivative of a degradation of current, obtained by placing a microwave absorber, DPPH, in the cavity. DPPH reduces the microwave field when it is at resonance so that part of the forward biasing from the microwave field is reduced and we see an absorption derivative of opposite phase to the SDR enhancement. (Fig 2)

The fractional enhancement of current is $\frac{\Delta I}{I_{Recomb}}$ The fraction of I that is recombination current is $\frac{I_{Recomb}}{I_{Recomb} + I_{diff}}$, thus the magnitude of the change decreases as the bias (V) increases and the diffusion current increases more rapidly than the recombination current.

The recombination current varies as $\exp(eV/\gamma kT)$ where in theory $\gamma = 2$ but experimentally is found to be 1.6. The diffusion current varies as $\exp(eV/kT)$, thus the ratio of recombination to diffusion current varies as $\exp\{(eV/kT)(1/\gamma - 1)\}$, i.e. decreasing with increasing V.

This predicts that $\frac{\Delta I}{I}$ falls off exponentially with bias, a relation which was found and is shown in figure 3. Variation of the gate voltage changes the geometry of the depletion region, giving different proportions of recombination and diffusion current, i.e. different γ and pre-exponential factors. By the application of a suitable gate voltage the depletion region can be extended under the gate to include the $Si-SiO_2$ interface, so that radiation-induced surface recombination centres contribute to the current. In our experiments we first investigated the dependence of surface recombination on the gate voltage. The devices were formed on the (100) Si surface and recombination centres were produced by irradiation with 20keV electrons. This treatment produced a maximum in the recombination current just before inversion of the n type Si. The irradiation increased the inversion voltage from about zero to $\simeq -20$ volts. The SDR signal was then investigated and the g value was found to be 2.008 ± 0.002. Present experiments are investigating in detail the dependence of the magnitude of the signal on gate voltage.

Spin Dependent Generation (SDG)

At negative bias voltages (< -5.0V) the reverse current of the p^+n gate controlled diode is dominated by the generation of electron-hole pairs in the depletion region. As with recombination, the gate voltage can be adjusted to include $Si-SiO_2$ surface centres in the depletion region.

The negative generation current is enhanced by the same microwave field and magnetic field that enhanced the forward recombination current. (Fig 4)

The SDG signal has opposite phase to the SDR signal since it is an enhancement of a negative current - DPPH degradation of the generation current is of opposite phase to DPPH degradation of recombination current. The size of the enhancement is about 5 parts in 10^4 (similar to the SDR enhancement). The g value and line width of the SDG and SDR signals are the same, indicating that the same type of recombination-generation centre is responsible for both signals. ($g = 2.008 \pm 0.002$). We are presently attempting to construct a model accounting for this surprising finding.

The effect of illumination was investigated by shining light from a bulb mounted near the sample. A dark forward current of $+2 \times 10^{-8}$A can be reduced, and made negative, by light-generated carriers, but the SDR signal is unaffected. Thus the number of carriers available does not limit the SDR signal, nor does band to band generation.

B-Field Dependence

By using different microwave frequencies, the B-field for spin resonance can be altered. The size of the signal is the same at 2500G (with a microwave frequency of 7GHz) and at 4220G (12GHz), supporting the field-independent model of Kaplan, Solomon and Mott. At the time of writing a signal has just been observed at 155 Guss (corresponding to a frequency of 440 MHz) of the same size as that at higher frequencies. Work is continuing on this topic.

4. PUBLICATIONS supported by this contract whose abstracts are reproduced in Section 2.

a) The Wigner glass and conductance oscillations in silicon inversion layers: M. Pepper and M.J. Uren, J Phys C 15 L617, 1982

b) Electron localisation and the 2D quantised Hall resistance: M. Pepper and J. Wakabayashi, J Phys C 15, L861, 1982

c) Spin-orbit coupling and weak localisation in the 2D inversion layer of Indium Phosphide: D.A. Poole, M. Pepper and A. Hughes, J Phys 15, L 1137, 1982

d) An experimental test of the scaling theory of conduction in two dimensions: R.A. Davis, M. Pepper and M. Kaveh, J Phys C 16, L2[illegible], 1983

e) Energy loss rate in silicon inversion layers: M.C. Payne, R.A. Davies, J.C. Inkson and M. Pepper, J Phys C 16, L291, 1983

f) Loss of dimensionality, localisation and conductance oscillations in n type GaAs FETs: D.A. Poole, M. Pepper and H.W. Myron, Physica 117B, 697, 1983

g) Electron localisation and the quantised Hall resistance in silicon inversion layers: J. Wakabayashi, H.W. Myron and M. Pepper, Physica 117B, 691, 1983

Figure Captions

1. The p^+n gate controlled diode used in this work.

2. The SDR signal ΔI is plotted as a fraction of the D.C. current $I_{D.C}$ against $I_{D.C}$ As $I_{D.C}$ increases with increasing bias so the SDR contribution decreases.

3. An example of an SDR signal with the DPPH signal used for calibrating the magnetic field. The magnitude of the DPPH signal has been reduced.

4. An example of the SDG signal, as with Figure 3 the magnitude of the DPPH signal has been reduced.

Fig. 1

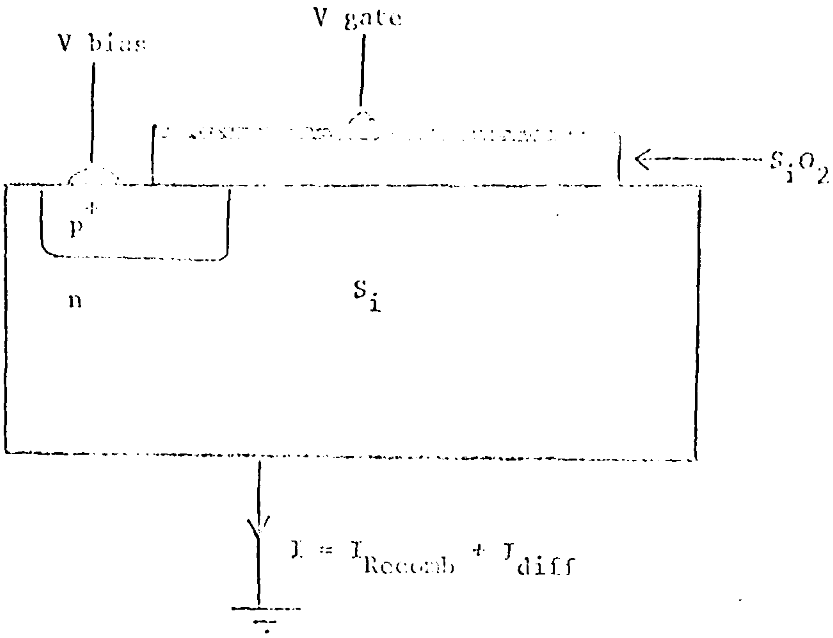
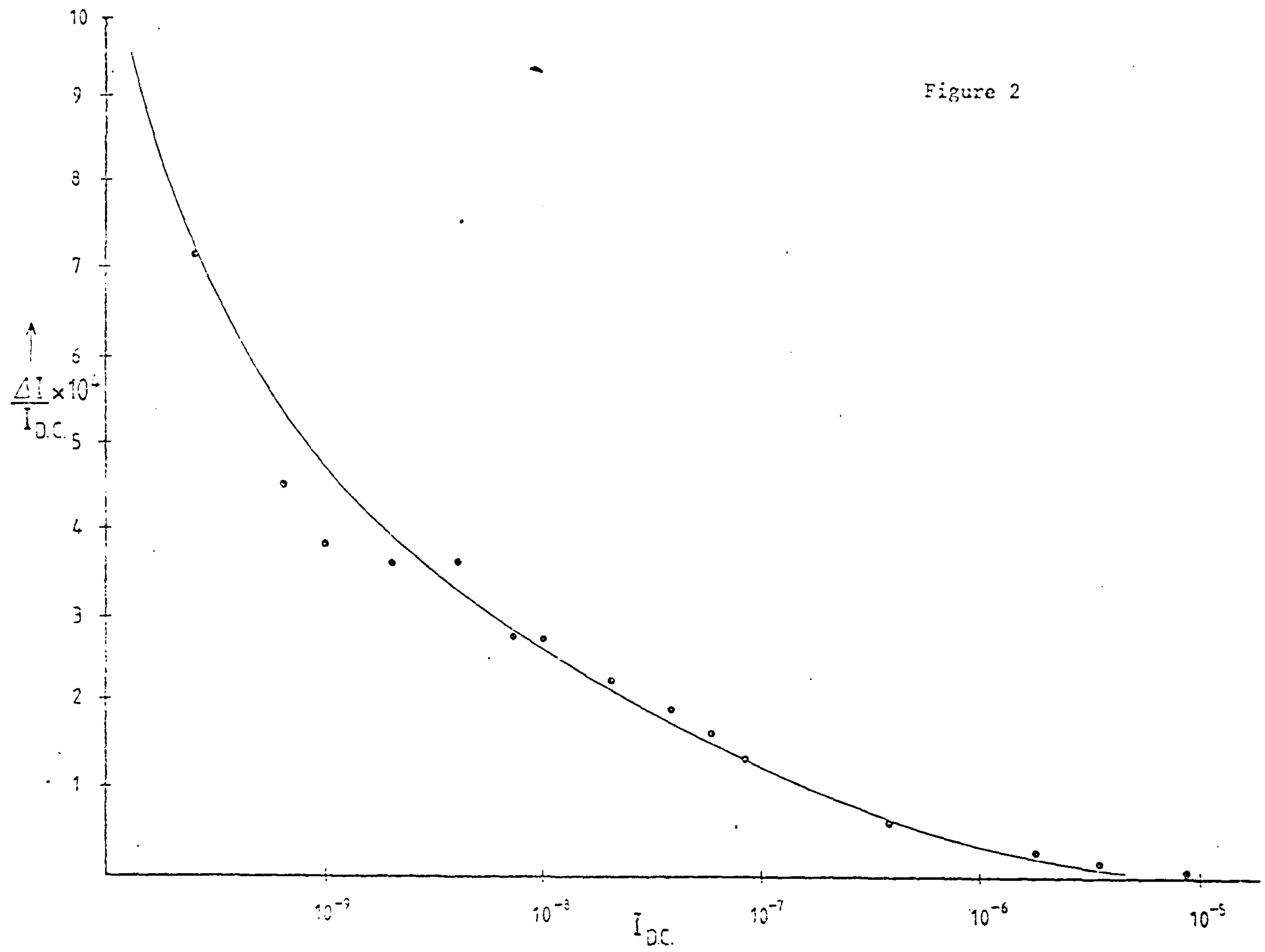


Figure 2



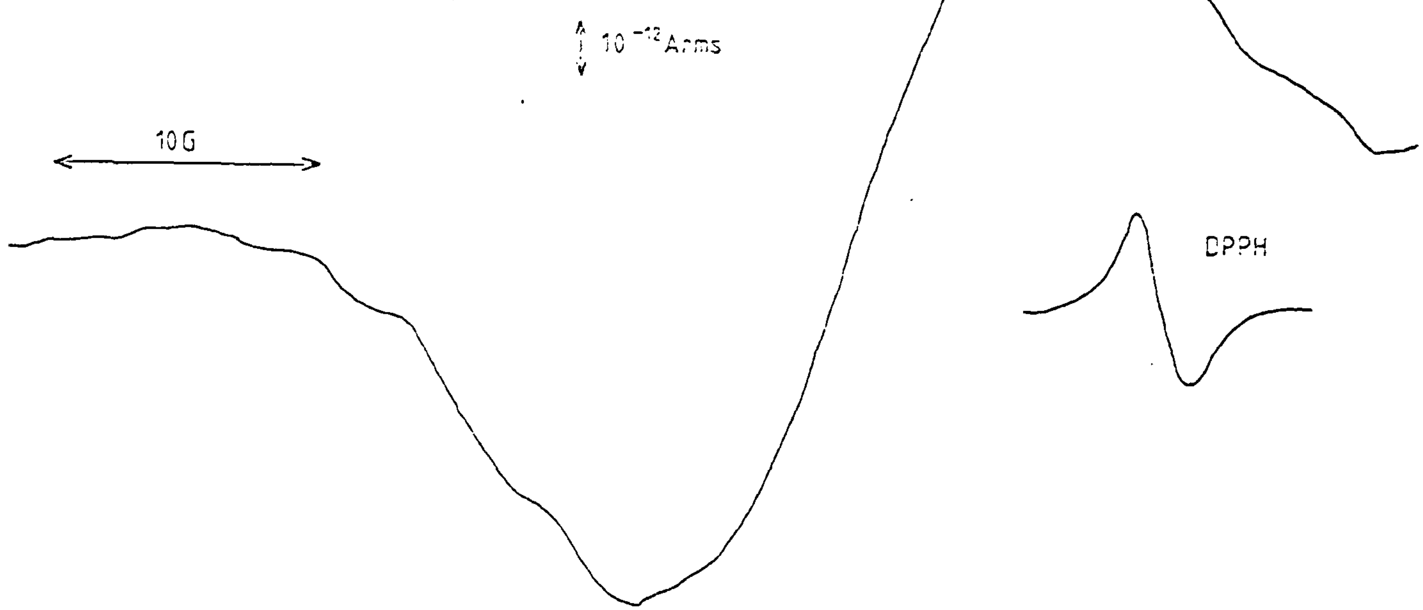
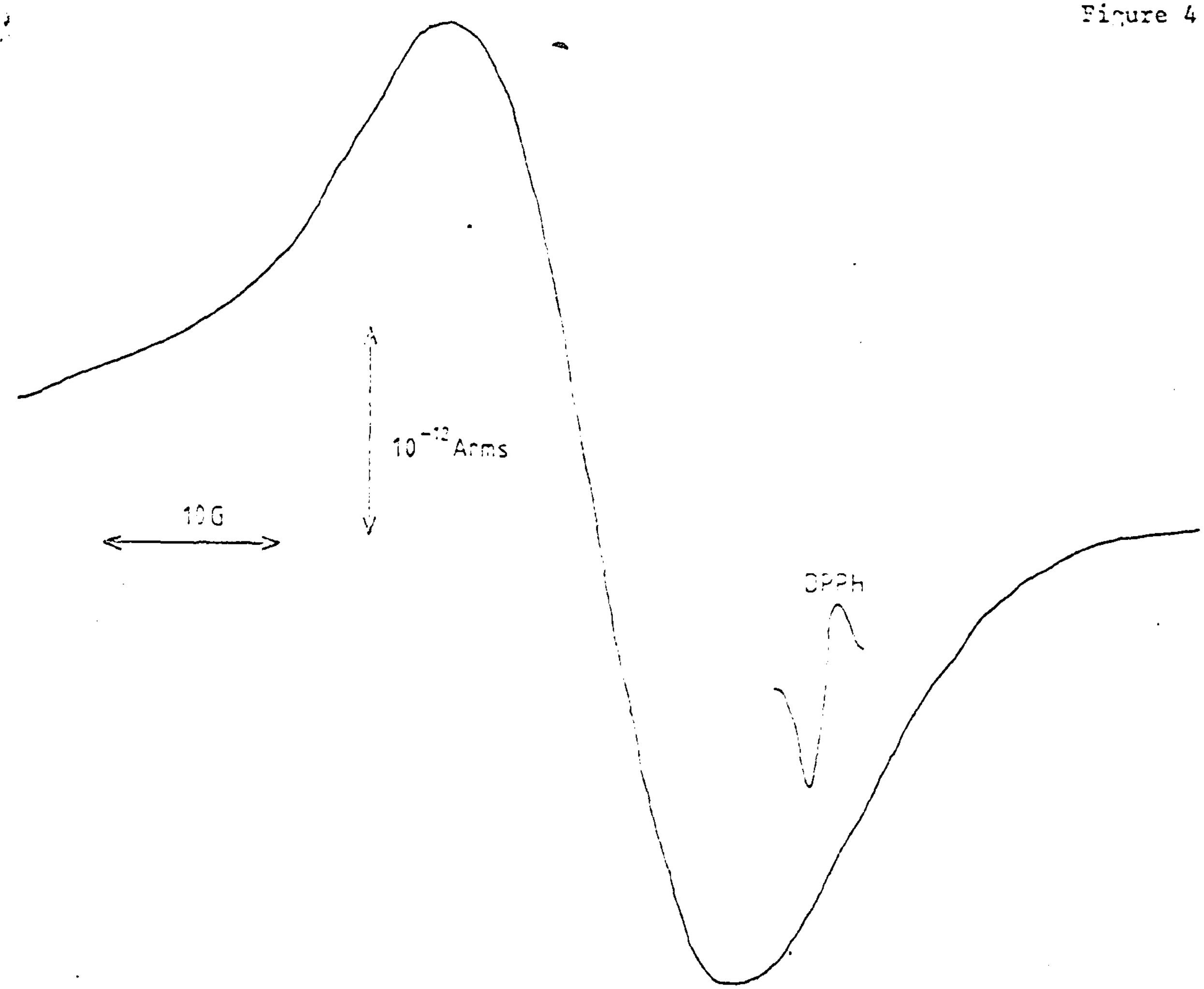


Figure 3

Figure 4



J. Phys. C: Solid State Phys., 15 (1982) L617–L625. Printed in Great Britain

LETTER TO THE EDITOR

The Wigner glass and conductance oscillations in silicon inversion layers

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Abstract. We show that on changing the nature of the background random potential the inversion layer of the Si MOSFET exhibits the conductance oscillations previously observed in both GaAs, the source and drain regions of Si MOSFETs and MOSFETs with a very high concentration of Na^+ ions at the Si–SiO_2 interface. Measurements of the temperature dependence of conductance show that oscillations are found when conductance is by an excitation process as well as hopping. The oscillations arise from an oscillating activation energy which is due to either an oscillating interaction contribution to the activation energy, or a negative effective density of states at certain values of carrier concentration. This appears due to electron ordering in a small correlating region and is contrasted with the case where the oscillations are absent, although localisation is due to both background disorder and the random field of localised electrons. It is shown that near the transition, localisation is due almost entirely to the random field of the localised electrons, and the system is a strongly interacting Fermi glass (or Wigner glass), even though the oscillations are not apparent.

It is now known that at finite temperatures there are corrections to the Boltzmann formulation of the two-dimensional (2D) metallic conductance. These corrections arise from the weak localisation of all states in 2D (Abrahams *et al* 1979, Gorkov *et al* 1979, Haydock 1981, Houghton *et al* 1980, Kaveh and Mott 1981), and cause the conductance of a 2D system in the 'metallic' range to vary as a power of temperature (Davies and Pepper 1982). An approximation to the power law, which is indistinguishable from it for small changes, is the now well known logarithmic correction (Dolan and Osheroff 1979, Bishop *et al* 1980, Uren *et al* 1980, Uren *et al* 1981a, b, Kaveh *et al* 1981, Davies *et al* 1981). It was first shown by Uren *et al* that interaction effects which give a similar correction can be separated by the effect of a magnetic field. However, in Si inversion layers the interaction contribution is only significant in the presence of a magnetic field and the principal mechanism determining the correction is that of weak localisation. This type of interaction effect is only present in the metallic regime and is not related to the effects discussed here.

Recent findings on the corrections do not entirely change the conclusions of earlier work on the transition between activated and metallic conduction (Mott *et al* 1975). The

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principal difference is that the mobility edge E_c is now seen as a localisation edge separating states which fall away exponentially from those which decay as a power law (Kaveh and Mott 1981). (Application of a magnetic field suppresses the power law localisation and turns E_c back into a mobility edge.) In this Letter we will be considering aspects of localisation where the corrections to metallic transport are not relevant. When the Fermi energy, E_F, is below E_c there are two parallel methods of conduction.

(i) Excitation to E_c where the conductance is given by $\sigma = \sigma_{min} \exp(-W/kT)$, $W = |E_c - E_F|$ and σ_{min} is the 2D minimum metallic conductance $0.1\, e^2/\hbar$ ($3 \times 10^{-5}\ \Omega^{-1}$). The possible dependence of σ_{min} on the form of the potential is discussed by Pepper (19[illegible]). The correction to σ_{min}, which is determined by the inelastic diffusion length of electrons at and above E_c, will be small when E_F [illegible] E_c as the diffusion length will be $(D\tau_i)^{1/2}$, D and τ_i being the diffusivity and lifetime of electrons at E_c. If the electron–electron scattering time τ_{ee} is shorter than τ_i then this will determine the diffusion length.

(ii) The variable-range hopping of electrons between localised states. In 2D ε varies as exp − (constant $T^{-1/3}$), and as the temperature is reduced this mechanism progressively dominates over excitation to E_c.

Information about the density of localised states can be obtained by analysis of the dependence of W on carrier concentration n. The density of states at E_F, $N(E_F)$, is $\sim dn/dW$, and analysis of experimental data shows that, if the total number of localised electrons is less than [illegible] 2×10^{11} cm^{-2}, $N(E_F)$ is always less than the free electron value. At E_c, $N(E_F)$ is in the range $0.5\, N(E)_{[illegible]}$ to $0.75\, N(E)_{[illegible]}$. If n is higher than 2×10^{11} cm^{-2} the value of $N(E_F)$ obtained from the analysis becomes higher than the unperturbed value. This cannot be correct and was interpreted as showing that E_c is rising as E_F increases because the random field of the localised electrons is itself increasing localisation (Pepper *et al* 197[illegible]). Examples of the analogous situation in 3D have been given by Mott (1979). If the role of the background disorder is small compared to the random field of the extra electrons the system can be described as a Wigner glass. By a Wigner glass we mean a Fermi glass where the disorder, which localises the electrons, is nearly but not entirely caused by other trapped electrons, as described by Mott (1979). We now give numerical evidence that the inversion layer at high levels of electron localisation is a Wigner glass at low temperatures.

The temperature dependence of conductance of an inversion layer with a high level of interfacial charge ($\sim 2 \times 10^{12}$ + charges cm^{-2}) has been measured in the temperature range 4.2 K–0.3 K. The activation energy in the regime where conduction is by excitation

Table 1.

Carrier concentration (electrons cm^{-2})	Activation energy W (meV)	$e^2/\varepsilon r$ (meV)	dn/dW eV^{-1} cm^{-2}	$A\left(\frac{e^2}{4\pi\varepsilon r}\right)$ (meV)	dE_c (meV)
10^{12}	0	18.3	$2.5 \times 10^{[illegible]}$	0.9	0.9[illegible]
9×10^{11}	0.04	17.4	$2.5 \times 10^{[illegible]}$	0.99	0.9[illegible]
8×10^{11}	0.08	16.41	$1.2 \times 10^{[illegible]}$	1.04	0.9[illegible]
7×10^{11}	0.16	15.32	8.0×10^{14}	1.17	0.87
6×10^{11}	0.29	14.2	5.0×10^{14}	1.2	
5×10^{11}	0.48	13.0	3.0×10^{14}	1.4	
4×10^{11}	0.8	11.6	1.2×10^{14}	2.41	
2.5×10^{11}	2.0	9.16			

to E_c was extracted as a function of n; these results are listed in the first two columns of table 1. The obtained $N(E_F)$ was considerably bigger than the value of $N(E)_{FREE}$ for the (100) Si surface (1.7×10^{14} eV^{-1} cm^{-2}), indicating that the electronic random field was effective in localising. We have estimated the change in the value of E_c, dE_c, from the relation

$$dW = dn/N(E_F) - dE_c.$$

Here $dn/N(E_F)$ is the movement of E_F. In order to extract dE_c from the observations we have to assume a knowledge of $N(E_F)$. Since the results are near the transition we assume $N(E_F)$ to be constant and equal to $0.6\,N(E)_{FREE}$ (Mott *et al* 1975). The values of dE_c obtained in this way are plotted in table 1. We also tabulate the change in the electrostatic repulsion between electrons when considered as point charges $\Delta(e^2/4\pi\varepsilon\varepsilon_0 r)$, where r is the mean distance between carriers; this is relatively insensitive to the form of distribution, and $\varepsilon = 8$ is the average dielectric constant at the Si–SiO$_2$ interface. It is clear that, near the transition, dE_c is near to the value of Δ. As the 2D localisation criterion for the tight binding case is $V_0/B \sim 1$, where V_0 is the RMS random potential and B is the bandwidth, the observation of $\Delta \sim dE_c$ indicates that E_c is moving up mainly due to the localisation of carriers by the random field produced by the localised electrons. This, then, in view of the approximate nature of the calculations, is strong evidence for the system being a Wigner glass, a conclusion not greatly changed if we allow the assumed value of $N(E_F)$ to alter within reasonable limits or assume a different expression for calculating the inter-electron separation. In these circumstances we would expect that the hopping conduction observed at very low temperatures is a multi-electron process as discussed by Knotek and Pollak (1974), Mott (1976) and Adkins (1978). The latter paper also considers a correlated conduction process at higher temperatures in which the mobility edge does not play as significant a role as envisaged here.

We now turn to the main subject of this Letter: the oscillations in conductance as a function of a carrier concentration shown by 2D systems at low temperatures. We will present our results on this phenomenon for Si inversion layers; these give strong evidence that the cause of the oscillations is the Coulomb interaction between electrons.

Previous investigations of the transport properties of electrons in a two-dimensional sheet in GaAs showed that when the carriers were localised the conductance oscillated as a function of carrier concentration (Pepper 1979). In this work the oscillations occurred when the distance between electrons was a multiple of a fundamental distance, indicating the electron interaction as a cause. (More recent work (Kitchen and Townsend 1982, Poole and Pepper 1982) indicates that the separation of the minima in the oscillations can be linear with carrier concentration.) As this behaviour was in the regime of strong localisation it appeared implicit that it arose from the contribution of the electron–electron contribution to the activation energy, being a maximum when the electrons are most ordered. However, it was not clear why electrons would attempt to order at certain values of carrier concentrations when the background potential arose from randomly distributed donors. Recently Dallacasa (1982) has attempted to remove this problem of the background by proposing that the ordering arises from the dielectric properties of localised electrons. On this model a whole series of order–disorder transitions occurs as the carrier concentration is altered, although, because of the existence of the random donor field, the ordered state will be a glass rather than a lattice.

Conductance oscillations have also been observed when conduction is in the accumulated surface of the source and drain regions of Si MOSFETs, and very weakly in inversion layers when a very high concentration of Na$^+$ was present at the Si–SiO$_2$

interface (Pepper *et al* 1979). Structure in the differential of conductance reported earlier may have a similar origin (Fang and Howard 1965, Pollitt *et al* 1976, Tidey *et al* 1974, Cole *et al* 1976). We now discuss the manner in which pronounced oscillations can be induced in the inversion layer by varying the form of the background potential.

The specimens used in this experiment were originally designed for investigating the transition from two to one dimensions, which will occur when the inversion layer is

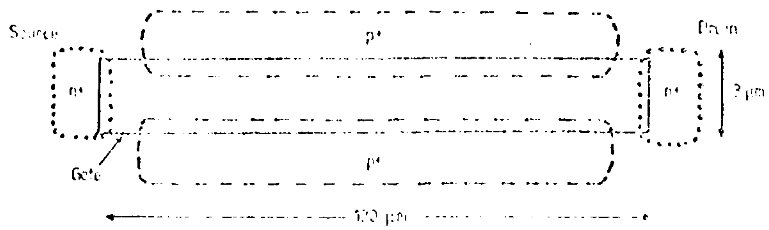

Figure 1. Schematic illustration of the device initially designed for the one-dimensional experiments and used in this work.

narrowed from the sides (Pepper 1981, Fowler *et al* 1982), as indicated in figure 1. The substrate is n-type (in this instance (100) orientation), and the channel is narrowed by applying a reverse bias to the p^+ regions. Alternatively, the acceptors from p^+ regions can be diffused across into the channel to give a doping gradient; this will cause the inversion layer initially to turn on in a line. The specimens used in this work had a lightly doped p-type layer at the surface and there was no evidence for the existence of a gradient in the doping. Some specimens showed structure of the type shown in figure 2; the oscillations could be moved by the application of a substrate bias, indicating that they originated in the channel. This result shows that the presence of impurity centres in the channel, changing the nature of the localising field, can induce the oscillations in the same way as impurity centres in the source and drain accumulation layers. We also found that strong oscillations could be induced in a device, which had not previously exhibited the effect, by avalanching the n^+p junctions at 4.2 K. The breakdown resulted

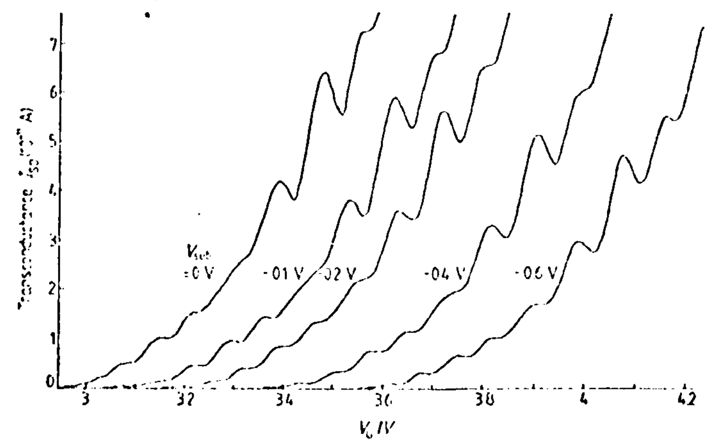

Figure 2. The oscillations in transconductance, as a function of substrate bias, in the devices. The temperature was 1.2 K and V_{SD} was 1.35 mV.

in hot electrons being injected over the 3.3 eV barrier at the Si–SiO_2 interface and subsequently trapped in the SiO_2 near the interface. After injection the threshold voltage became more positive, indicating that ~ 5×10^{11} cm^{-2} electrons were trapped in the oxide; the mobility (for non-localised electrons) was reduced by ~40% and, at low values of carrier concentration, large conductance oscillations were present although a clear periodicity was not apparent (cf figure 3). The application of a substrate bias

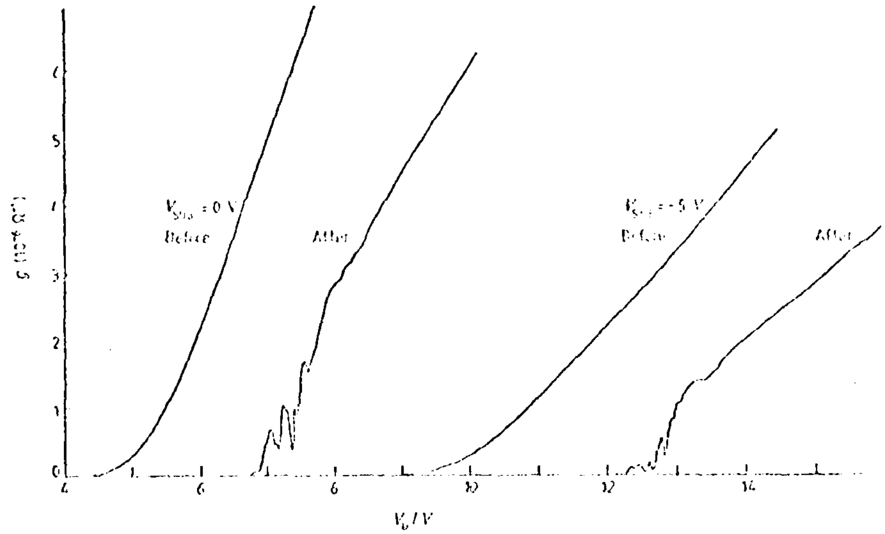

Figure 3. The conductance oscillations induced by avalanche injection into the SiO_2. We display both 'before' and 'after' and the effect of substrate bias. The temperature was 4.2 K and V_{SD} was 1.0 mV.

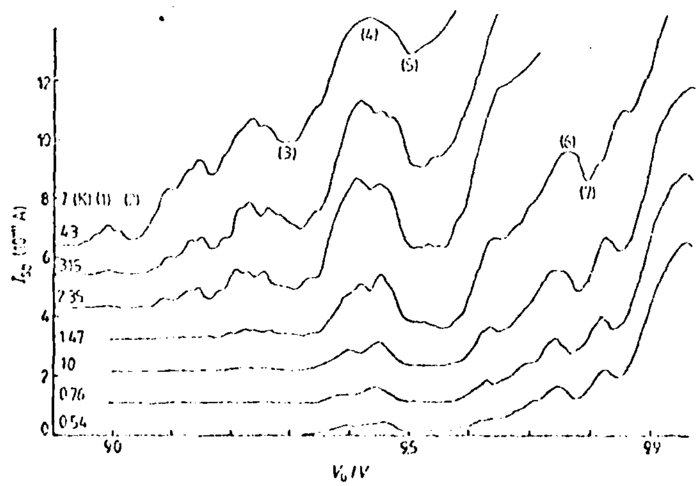

Figure 4. The temperature dependence of the oscillations in a structure subjected to avalanching. V_{SD} was 0.1 mV. The numbers indicate the location of E_F used in figure 5.

altered the dependence on the gate voltage of the oscillations, confirming that they were caused by electrons in the inversion layer. As expected, the structure sharpened with decreasing temperature and new oscillations appeared (figure 4); for example, the features identified as 6 and 7 in figure 4 were a 'shoulder' until they split into a maximum and a minimum below ~2 K. Since the activation energy was ~0.2 meV the resolution

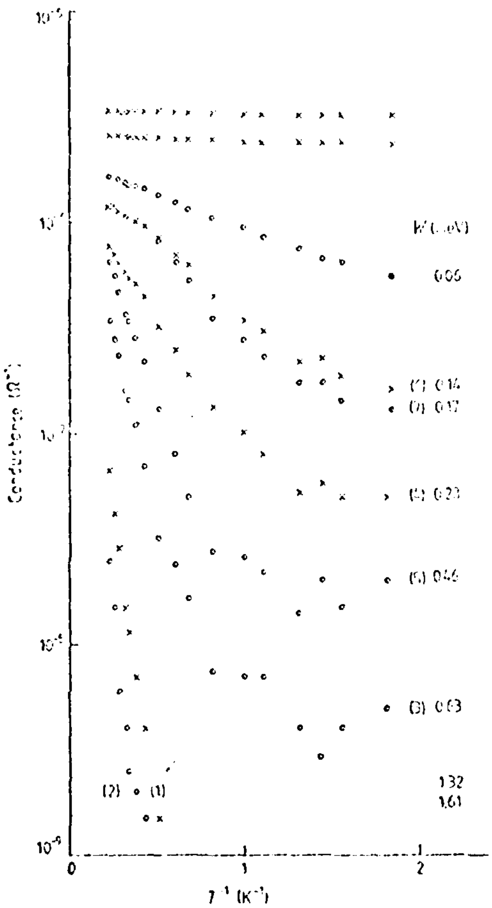

Figure 5. Detailed temperature dependence of conductance of the structure shown in figure 4; the location of the Fermi energy is indicated by the numbers (1, 2 . . . 7) in figure 4, $V_{SD} = 0.1$ mV. The activation energies are indicated for the maxima 1, 4, 6 and minima 2, 3, 5 and 7. Crosses denote maxima; circles denote minima.

was clearly due to the absence of thermal smearing. Increasing the gate voltage caused the oscillations to disappear when the conductance became roughly independent of temperature (the 2D localisation and interaction corrections were not investigated in this work). The detailed temperature dependence of the oscillations in figure 4 is shown in figure 5. As seen, at the higher temperatures the results fit the law $\sigma = \sigma_0 \exp(-W/kT)$, regardless of whether σ is a maximum or minimum. At low

temperatures the points leave the extrapolated plot in the manner observed when 'hopping' determines the conductance. The scatter of points and initial increase of conductance in the regime is not error but is reproducible and is also found for oscillations in GaAs, as is the very weak dependence on temperature at the lowest temperatures.

It is striking that the behaviour $\sigma = \sigma_0 \exp(-W/kT)$ is identical to that found in conventional inversion layers, except that in the latter case W decreases smoothly with increasing n and $\sigma_0 = \sigma_{\min}$ ($\approx 0.1\, e^2/\hbar$). (A local maximum in activation energy due to Na^+ impurities occurs when conduction is by hopping and not by excitation (Hartstein and Fowler 1978)). We can equate σ_0 with $\sigma_{\min}$ if the aspect ratio of the current limiting region is 0.1. In order to investigate this region, in which resides the dominant portion of the injected oxide charge, the Shubnikov–de Haas effect was measured. The threshold voltage found from the measurements of the Shubnikov–de Haas oscillations had the pre-avalanche value, indicating that only a small length of the channel was affected by the oxide charge and therefore limited the current. Thus, this result supports the view that the aspect ratio of the current limiting region is considerably greater than that of the device (0.0[illegible]5), and the value of 0.1 required to equate σ_0 with $\sigma_{\min}$ is not unreasonable.

If the oscillating activation energy were only observed in the hopping regime, then a number of explanations based on an oscillating density of states would be possible. This type of behaviour could arise from one dimensional behaviour or percolation aspects of transport. However, the oscillating behaviour of an excitation energy cannot be explained without reference to the Coulomb gap (or Coulomb contribution to the activation energy), and appears related to the earlier result of a Coulomb contribution to $E_c - E_f$ (Pepper *et al* 1974). The observation of the oscillations at lower temperatures in GaAs (Poole and Pepper 1982 to be published) where conduction is by hopping shows that the same interaction is present. The change in the activation energy which occurs on going from conductance maximum to minimum is always greater than the change in $e^2/4\pi\varepsilon\varepsilon_0 r$ for these two values of carrier concentration. However, this change is always much less than the mean value of $e^2/4\pi\varepsilon\varepsilon_0 r$ and so can be satisfactorily accounted for by enhanced ordering of the electron distribution; we do not have a suggestion as to the cause of the ordering other than that of Dallacasa (1982). An alternative description of the effect is a negative effective density of states postulated by Bello *et al* (1981) for Wigner condensation in 2D (also Chenskii and Tkach 19[illegible]). However, this treatment does not yield a series of oscillations as found in the experiments.

From the experimental point of view it seems that the electron–electron interaction produces three different effects, which are related. Initially, with a very low electron concentration, the system is a non-interacting Fermi glass. As the electron concentration increases, depending on the type of potential fluctuations, there are three possible consequences.

(i) Interactions reinforce disorder and there is a smooth progression to the interacting Fermi glass, or Wigner glass, in which localisation is due to disorder plus interaction. The localisation energy, $E_c - E_F$, decreases to zero smoothly with increasing n, although the effect of the interaction is to cause E_c to rise.

(ii) The type of random potential allows a measure of ordering to occur due to the strong electron electron interaction. Localisation is due to disorder and the Coulomb interaction but sharp changes in $E_c - E_F$ occur at certain critical values of

carrier concentration giving rise to the conductance oscillations. The work reported here suggests that this occurs when the current limiting region is small.

(iii) If the background disorder is sufficiently weak then the number of carriers localised is small ($\sim 2 \times 10^{11}$ cm^{-2}) and the Coulomb interaction does not appear to play a role. However, on the metallic side of the transition a degenerate electron gas with low n is observed and here Coulomb effects may be significant, particularly in the presence of a magnetic field (Wilson *et al* 1981).

We have also found that oscillations can be induced when positive charge, in the form of trapped holes, is created in the SiO_2 near the Si–SiO_2 interface by electron beam irradiation. The oscillations are most pronounced when a narrow line of charge is created and are weaker when the whole device area is irradiated (this result has also been found by R A Davies, private communication). Similarly we have not found oscillations when an inversion layer is formed on a highly doped substrate, whereas they can be found in the diffused regions and eliminated by an applied voltage of a few microvolts, indicating a small current limiting region.

In the experiments reported here, the pre-exponential factor σ_0 is approximately constant from strongly activated to just metallic behaviour. This indicates that the whole current limiting region gives rise to the oscillations and is not changing its size as the gate voltage changes, and that this region is contributing uniformly to the effect. It is apparent that the effect is only strongly observed when a statistically small number of electrons is present. This may be due to a small length over which the electron behaviour is coherent, and as this length increases the oscillatory behaviour averages to zero. Increased size of the current limiting region may be the cause of the absence of any clear periodicity of the oscillations. This may be a fundamental physical limitation or a reflection of the variation in background charge over a larger area. The critical interplay of disorder (due to oxide charge) is illustrated by differences in the oscillatory behaviour between chips on the same silicon wafer, and also by differences induced by thermal cycling between room temperature and 4.2 K.

The results and discussion presented here concentrated on the localised regime. However, the effect has now been observed in a weaker form in the metallic regime in GaAs. Further details will be reported shortly (Poole and Pepper 1982 to be published).

We have enjoyed many discussions with Professor Sir Nevill Mott, Dr P Townsend and Mr V Kitchen of Essex University and Mr D A Poole. This work was supported by the SERC and, in part, by the European Research Office of the US Army. M J Uren possessed an SERC CASE Studentship with the Plessey Company and the work was performed when M Pepper was on leave of absence from the Plessey Company.

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LETTER TO THE EDITOR

Electron localisation and the 2D quantised Hall resistance

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Abstract. Using Si inversion layers we have investigated the plateaux of quantised Hall resistance [illegible] a temperature dependent [illegible] at the level [illegible] the centre of the level [illegible] the plateau. [illegible] substrate [illegible] present, is weak. Under these circumstances the localisation is long range and can be interpreted as the absence of extended states [illegible].

Measurements under AC conditions reveal the appearance of a plateau with increasing frequency when one is not apparent at DC. This is discussed in terms of the localisation length and it is suggested that electrons behave as if delocalised when the frequency is such that the drift length is less than the localisation length.

The behaviour of ρ_{xy} throughout the ground Landau level has been analysed and it is shown that a 'normal' Hall effect is not obtained from extended states at the centre of the level. This is in agreement with theories which suggest that the extended state σ_{xy} compensates for the presence of localised states in the tail of the level. It is found that when extended states are present in the second level (0 ↓ −) they not only compensate for the localised states in this level but also compensate for the bottom level (0 ↑ +) which is entirely localised. On the other hand, when states near the centre of the level are localised, and a plateau is not found, the results indicate that weakly localised carriers contribute normally to the Hall effect. The absence of any effect of the weak localisation on σ_{xy} is the same as has been found for weak, non-magnetic localisation in both two and three dimensions.

The Shubnikov–de Haas effect exhibited by a two-dimensional electron gas has been a topic of investigation since the initial work on Si inversion layers (Fowler *et al* 1966). Recently interest has developed in the Hall effect when the Fermi energy E_F is in the density of states minima between Landau levels. Wakabayashi (1978) and Wakabayashi and Kawaji (1980) showed that the Hall conductivity deviated from theory in these regions and formed a plateau independent of carrier concentration. Subsequently Von Klitzing *et al* (1980) found that the value of Hall resistance in the pleateau regions was

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quantised in units of h/e^2 to an extremely high accuracy. This has given rise to the possibility that the quantised Hall resistance can be used a standard for the accurate determination of the fine structure constant.

From an experimental point of view σ_{xx} has to be vanishingly small for the observation of the plateau in Hall voltage. If this is the case, the Hall conductivity σ_{xy} is given by $\sigma_{xy} = ne/B$, where n is the electron concentration required to fill the Nth Landau level. Correction terms are [illegible] of σ_{xx} [illegible] Localisation in the tails of [illegible] Landau levels [illegible] has been known for some time (Kawaji and Wakabayashi 1977, Nicholas *et al* 1978, Pepper 1978). Depending on the density of states at E_F, $N(E_F)$, conduction is either by an excitation process or [illegible] hopping. Our analysis of the [illegible] hopping [illegible] indicates that the localisation length can be several thousand ångströms. This long length is observable because the low value of $N(E_F)$ [illegible] scattering rate.

Recent theoretical [illegible] 2D [illegible] are not relevant here, as it has been demonstrated that a magnetic field suppresses this type of weak 2D localisation (Uren *et al* 1980, Uren *et al* 1981). Similarly the electron–electron interaction is not important as this does not affect σ_{xy}, only σ_{xx}, which, in the region of the Hall plateau, is virtually zero.

Recent theoretical work [illegible] the effect follows on from Ando *et al* (1975) in suggesting that the contribution of extended states of σ_{xy} exactly compensates for any decrease caused by localised states in the tails of the levels (Prange 1981, Laughlin 1981, Aoki and Ando 1981). Thus these theories suggest an additional requirement for the observation of the quantised plateau, namely the presence of extended states below the Fermi level. In this work we have investigated the underlying physics of the quantised Hall resistance and show the relationship between the appearance of the quantised plateau and the localisation of carriers.

The specimens used were conventional Hall and Corbino geometry Si MOSFETs fabricated on the (100) surface with the same preparation treatment. The Hall devices were 400 μm long and 50 μm wide; two pairs of Hall probes were located equidistantly along the channel. The Corbino device possessed a 25 μm long channel and an aspect ratio of 26. In both types of device the oxide was about 1200 Å thick. Measurements were AC method and were performed in a magnetic field of 9 T. The experiments were concentrated on the Hall plateau appearing between the spin split levels of the ground Landau level, denoted by $(0\uparrow -)$ and $(0\downarrow +)$. ($\uparrow$ and $\downarrow$ refer to spin parallel and antiparallel; $+$ and $-$ refer to the lower and higher valleys, the degeneracy of which is lifted at these fields). Localisation of electrons is particularly strong in the lowest spin-split levels and consequently the plateau formation can be studied as the degree of localisation is altered.

We first illustrate the effects of increasing temperature on the quantised Hall plateau between the 0 and 1 Landau levels. The electrons were heated by increasing the current I_x through the device, and, as seen in figure 1, increasing I_x results in a decrease in the region where dV_H/dI_x is constant, and eventually the plateau region disappears. When E_F is in the plateau region, electrons travel through the inversion layer without scattering except near the end contacts where the equipotentials bunch together. There is thus no energy dissipation except near the contacts and consequently there is no field-produced electron heating. The temperature of the electrons is raised by Joule heating near the contacts. The removal of the plateau as the temperature is increased is expected on the basis of extended state conduction and arises because σ_{xx} is no longer vanishingly small when the temperature is increased. In figure 2 we show the temperature and electric

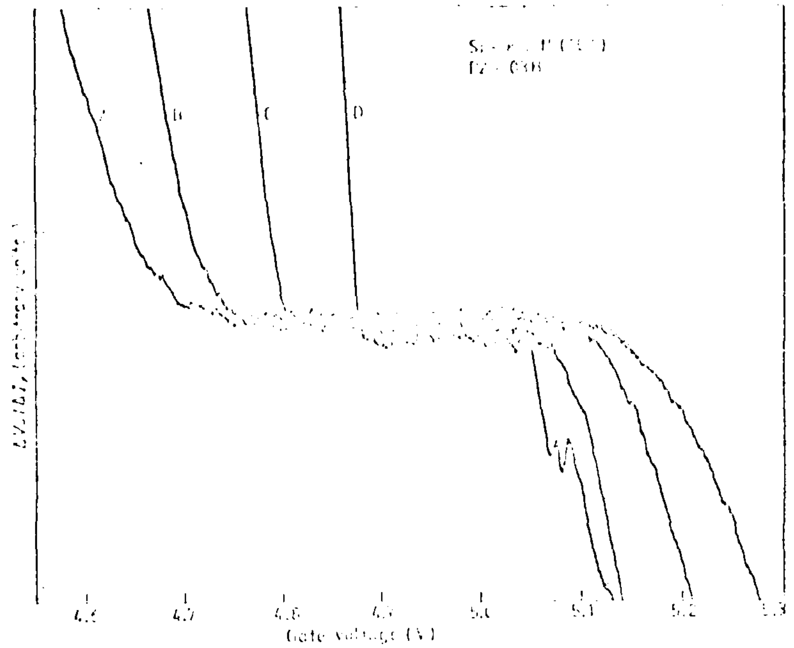

Figure 1. The current dependence of the plateau width between 0 and 1 [illegible] levels illustrated by plotting [illegible] versus gate voltage. T = 1.2 K; H = 9.0 T; f = 180 Hz. The AC component of current is kept at 0.025 μA and DC components are: A, 0; B, 2; C, 5; D, 10 μA.

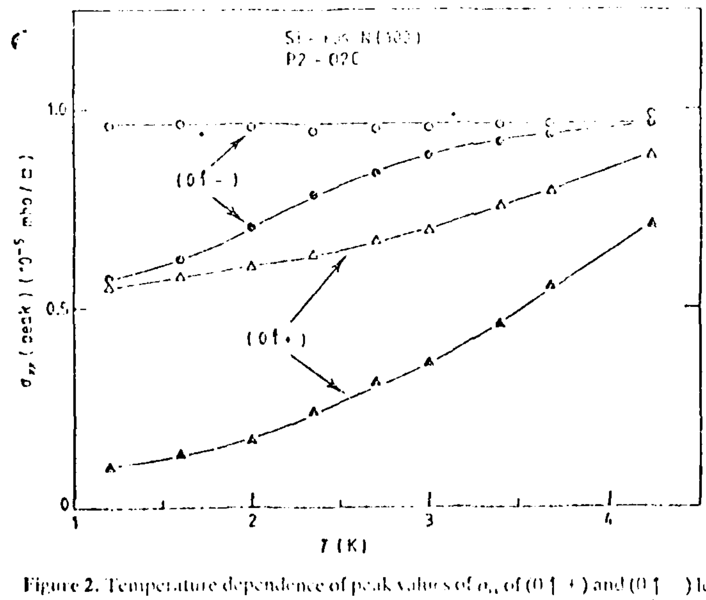

Figure 2. Temperature dependence of peak values of σ_{xx} of (0 ↑ +) and (0 ↑ −) levels at two different values of source–drain field. Full symbols, E_{SD} = 0.02 V cm^{-1}; open symbols, E_{SD} = 4.0 V cm^{-1}. B = 9 T.

field dependence of σ_{xx} measured directly with a specimen of Corbino geometry. As is clear, for low source–drain field the peak value of σ_{xx} in the two $0\uparrow$ levels decreases with decreasing temperature, indicating that the entire level is localised. The temperature dependence of σ_{xx} was not analysed in detail in this work, but it is clear that increasing the electric field achieves delocalisation in the $(0\uparrow-)$ level. Figure 3 shows ρ_{xx} and ρ_{xy} measured directly using a Hall geometry device. We now discuss two aspects of these figures.

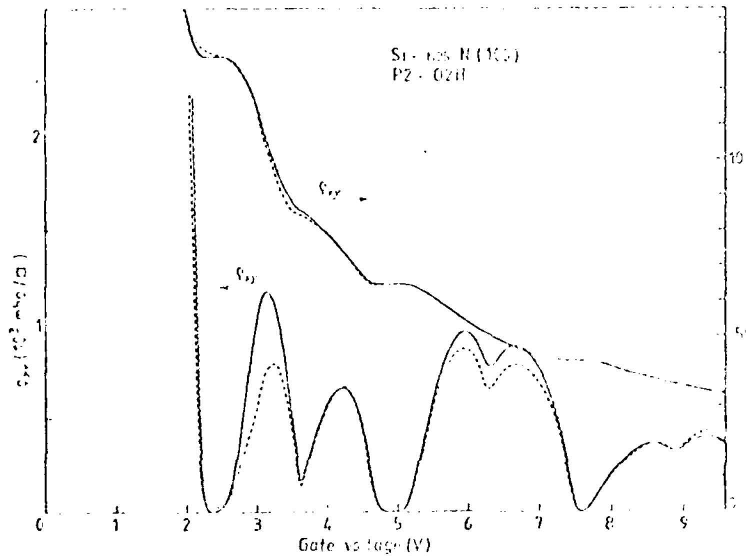

Figure 3. The behaviour of ρ_{xx} and ρ_{xy} in the 0 and 1 Landau levels at two different values of drain current. Broken curves, 0.05 μA; full curves, 1.0 μA. $T = 1.2$ K; $H = 9.0$ T; $f = 180$ Hz.

First, increasing the current (temperature) produces the opposite effect on ρ_{xy} between $(0\uparrow-)$ and $(0\downarrow+)$ compared to between 0 and 1. When E_F is between 0 and 1 increasing I_x results in a decrease in the width of the plateau. However, when E_F is between $(0\uparrow-)$ and $(0\downarrow+)$ increasing I_x results in the formation of a plateau. This is also illustrated in detail in figures 4(a) and (b); it is seen in figure 4(a) that a plateau exists at 2.4 K and then collapses as the temperature is lowered to 1.5 K. It is clear that decreasing the temperature results in the localisation becoming apparent through a decrease in conductivity and the disappearance of the plateau. Figure 4(b) shows that once the plateau is formed in this way it can then be removed again by further increasing the current in exactly the same manner as the removal of the plateau between 0 and 1. The subsequent removal of the plateau is not related to the localisation but is due to the increase in σ_{xx}.

The second feature of the figures is the presence of the plateau in the spin gap when there are extended states in the $(0\uparrow-)$ level, but the entire $(0\uparrow+)$ level is localised and shows temperature-dependent conduction. This indicates that the extended states in $(0\uparrow-)$ are compensating not only for the localised states in this level but also for the $(0\uparrow+)$ level. This suggests that the compensation mechanism is more general than has

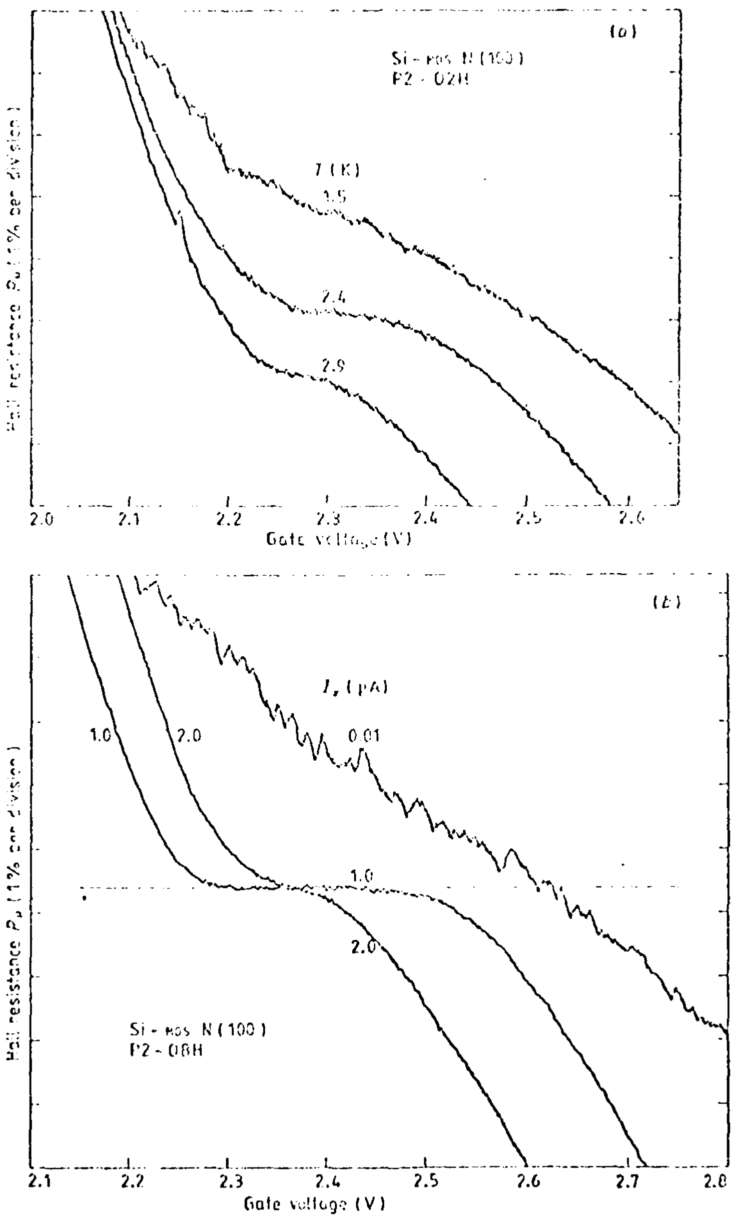

Figure 4. (*a*) Temperature dependence of plateau region between (0 ↑ −) and (0 ↓ +) levels (I_s = 0.05 μA); (*b*) Current dependence of plateau region between (0 ↑ −) and (0 ↓ +) levels (T = 1.2 K). H = 9.0 T; f = 180 Hz.

been suggested. This has also been observed in the Hall current experiment (Kawaji and Wakabayashi 1981), where the lowest level shows activated behaviour and there is no plateau in the gap region between (0↑+) and (0↑−) but between (0↑−) and (0↓+).

The ease with which the localisation can be changed in these specimens is evident from the creation of a plateau by the application of a substrate bias of −2.0 V. The decrease in localisation caused by the bias is also evident from an observed increase in σ_{xx} at the centre of the level. In the absence of a magnetic field similar delocalisation effects are observed (Pepper 1977) and have been related to the decreasing length scale of the potential fluctuations; it is not clear if this is the appropriate mechanism in these

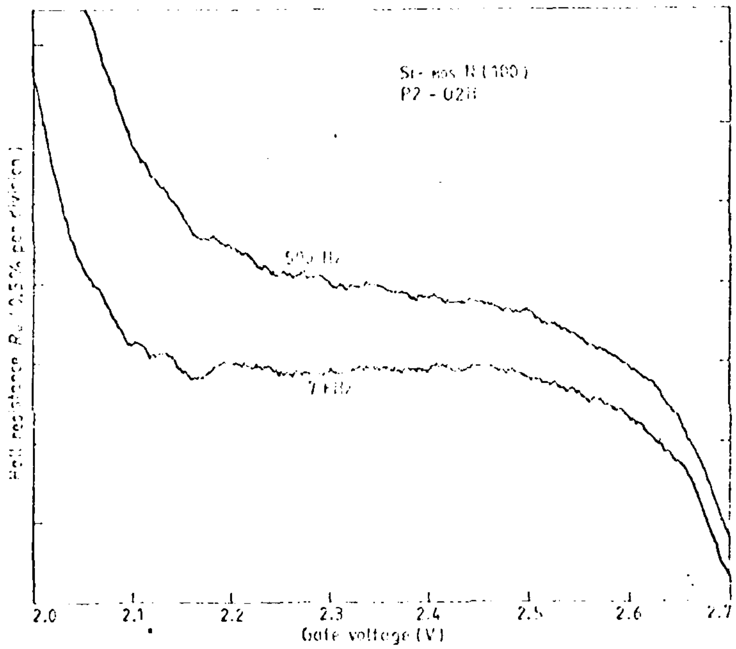

Figure 5. Frequency effect on plateau formation between (0↑−) and (0↓+) levels. T = 1.2 K; H = 9.0 T. The current is 0.25 μA, which is sufficiently low that a plateau is not observed at low frequencies or DC.

experiments. (These effects are not related to the observation of the true inversion layer conductance when a substrate bias is applied to a contact limited device (Wilson *et al* 1981). Transport when dominated by localisation in the contacts can be mistaken for inversion layer localisation (Tsui and Allen 1975).

These results give strong support to theories suggesting that extended states must be present for the observation of the quantised plateau. However, when AC measurements are performed a plateau can be observed even when all states are localised and a plateau is not found under DC conditions. This is shown in figure 5 where, at low frequencies and low values of current, a plateau is not present when E_F is between (0↑−) and (0↓+). Increasing the frequency results in the progressive formation of the plateau, a process

evident at as low a frequency as 7 kHz. However, due to capacitive loss in the device the accuracy of the Hall resistance cannot be estimated to better than 3% at this frequency. We suggest that the frequency effect arises because the velocity of the centre of the orbit is determined solely by the current and magnetic field being zero for zero current.

In the plateau region the drift velocity of the electrons, V_D, is given by

$$V_D = E_H/B$$

where E_H is the Hall field, equal to 7 m s^{-1} at $I_x = 0.25$ μA. If the states are weakly localised then we may use this value of V_D if

$$1/\alpha > 2V_D/\omega$$

where α is the decay constant of the wavefunction and $\omega/2\pi$ is the frequency of the applied field; this is essentially the condition for the localisation to become unimportant. The drift velocity V_D is used rather than the diffusivity D because, under these conditions, the electron is not scattered and D is not meaningful. The effective delocalisation condition can be met at a frequency of ~7 kHz and a current of 0.25 μA corresponding to a value of $1/\alpha$ ~ 300 μm. This implies that the increasing temperature increases the localisation length until, when this was equal to the specimen length, an effective delocalisation was achieved. This extremely long localisation length is only observable because electrons in the centre of the Landau level are not scattered when E_F is in the band tail. It is not observable when E_F lies in these states because of the rapid rate of inelastic scattering. It should be noted, however, that when we take into account the compensation effect on the drift velocity V_D the calculated localisation length increases. As discussed earlier, if the extended states in the $(0\uparrow-)$ level are compensating not only for the localised states in this level but also for the $(0\uparrow+)$ level, the drift velocity, and hence the localisation length, effectively doubles. However, the present experiment does not yield an accurate frequency for the onset of the plateau, and so we cannot quantitatively discuss the localisation length.

At the lowest values of current (~10^{-9} A) the effect of increasing the frequency was difficult to establish because of the noise. Consequently it was not possible to examine the frequency dependence when the current was not effective in delocalising carriers. It was noted that when the plateau was present, it tended to narrow slightly with increasing frequency. Our previous measurements of conductance in the minimum of $N(E_F)$ (Wakabayashi and Pepper 1982) showed that the change in σ_{xx} with increasing frequency was small and could not account for this narrowing of the plateau. Thus we attribute the effect to the progressive removal of the consequences of the localisation of states in the Landau level tail as the frequency increases. The states in the tail will be more tightly localised than states in the centre, and consequently higher frequencies are required to remove the effects of localisation. Capacitive loss problems with large geometry devices prevented us from investigating frequencies above 10 kHz, and so these effects could only be observed when the electron temperature was increased to a value such that the DC plateau was on the verge of appearing.

The results of the frequency dependence are in qualitative agreement with our results on the temperature dependence, namely that extended states or states which behave as extended are required below E_F for plateau formation. This frequency effect can only be found at low frequencies because the virtual removal of scattering when E_F is in the minimum of $N(E)$ results in a long inelastic scattering time and the wavefunction is coherent over large distances. Because the localisation is very weak the behaviour is

quite different to that in regimes of strong localisation. In this context strong localisation would correspond to a cyclotron orbit bound by an impurity potential, as suggested in earlier numerical work (Aoki 1977) and recent numerical experiment (Ando 1982) showing the existence of a mobility edge in a Landau level. The localisation near the centre of the level is then due to comparatively long range fluctuations in potential and regarded as potential barriers separating regions where the wavefunction is extended below the Fermi level. States in the tails of the levels are localised by the short-range potential. The role of temperature in delocalising is to remove the barrier by expanding the bandwidth of extended states. This means the mobility edge, E_c, moves away from the centre of the level with increasing temperature. This temperature dependence of E_c has a number of possible origins: such as screening, the temperature dependence of the Coulomb contribution to the localising potential and thermal excitation.

If the existence of a conduction path through the specimen is used as the criterion for the presence of extended states then the frequency effect can be explained by the localisation length being the effective specimen length. As soon as the AC drift length becomes less than the localisation length of states below E_F the plateau should appear. Similarly when the AC drift length of electron at E_F becomes less than the localisation length of these electrons the plateau should disappear again. We note that if most states in the level are weakly localised a plateau will not be found when E_F lies in these states because of the high value of σ_{xx}. However, σ_{xx} will decrease rapidly with decreasing temperature when E_F is in weakly localised states giving rise to a rapid increase in plateau length. Such a phenomenon has been found recently in GaAs/GaAlAs (Paalanen *et al* 1982).

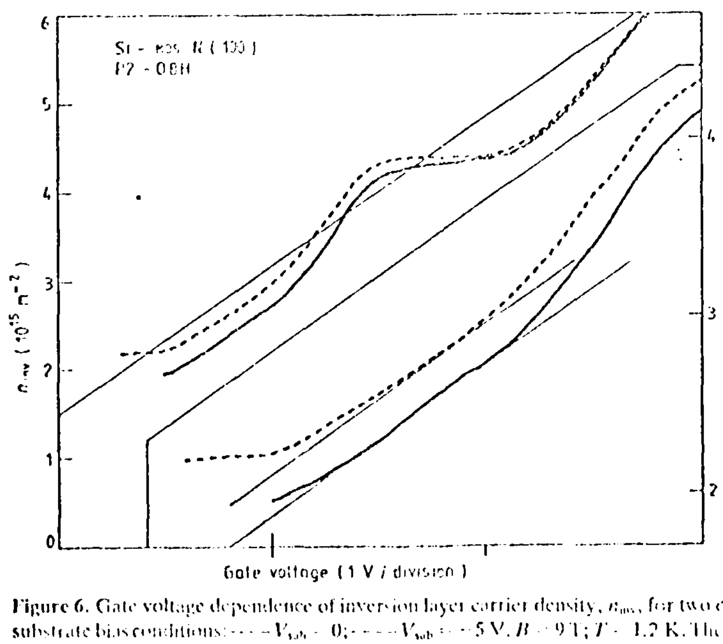

Figure 6. Gate voltage dependence of inversion layer carrier density, n_{inv}, for two different substrate bias conditions: --- V_{sub} = 0; - - - V_{sub} = −5 V. B = 9 T; T = 1.2 K. The plateau is observed when V_{sub} = −5 V. The straight line is the carrier concentration obtained from $n_{inv} = C_{ox}(V_g - V_T)/e$, where the threshold voltage V_T was determined from the period of the Shubnikov–de Haas oscillation in the standard way.

We now consider the behaviour of the Hall effect throughout the Landau level. This behaviour is particularly relevant in view of the suggestions that the presence of the plateau alters the behaviour of σ_{xy} when states at E_F are extended. Our procedure is to calculate the number of inversion layer carriers, n_{inv}, from the Hall effect and compare this quantity to the real number given by $C_{ox}(V_g - V_T)/e$, where V_g and V_T are the values of applied gate voltage and threshold voltage respectively. We calculate n_{inv} from the relation

$$n_{inv} = -(B/e)(\sigma_{xx}^2 + \sigma_{xy}^2)/\sigma_{xy} = B/e\rho_{xy}.$$

As states in the tails of Landau levels are localised the value of n_{inv} obtained near the centre of the level is not expected to equal $C_{ox}(V_g - V_T)/e$. However, the rate of change of n_{inv} with V_g should be simply C_{ox}/e, provided the extended states are giving rise to a 'normal' Hall effect. In figure 6 we show n_{inv} plotted against V_g for two different substrate bias conditions and a corresponding presence and absence of the plateau in the spin gap. It is seen that, near the centre of the Landau level, n_{inv} is below the true carrier concentration, and then rises above it as V_g is increased. The line indicating the real value of carrier concentration passes through the centre of the plateau. However, the rate of change of n_{inv} with V_g in the centre of the level is given by C_{ox}/e when the plateau is absent. When the plateau region is present this linearity in n_{inv} with V_g is absent. This is evidence that the presence of the plateau region distorts the value of σ_{xy} throughout the region of extended states near the middle of the level. Because of this distortion the rate of change of n_{inv} with V_g is always different to the real value. However, when states in the centre of the level are weakly localised the plateau is not present and now the rate of change of n_{inv} with V_g is correct. The same difference in behaviour between 'plateau' and 'non-plateau' case is found when we calculate n_{inv} using the simplified quantum expression developed by Ando *et al* (1975). Here

$$\sigma_{xy} = -n_{inv}e/B + (\Gamma/\hbar\omega_c)\,\sigma_{xx}$$

where Γ is the width of the Landau level given by $\Gamma = (2\hbar^2\omega_c/\pi\tau)^{1/2}$, where ω_c is the cyclotron frequency and τ is the scattering time in the absence of magnetic field.

Thus, in summary, these results agree with theories that the extended state σ_{xy} compensates for the presence of localised states in the tail of the level. When states at the centre of the level are weakly localised the plateau of quantised Hall resistance is not present when E_F is in the tail of the level and the compensation effect on σ_{xy} is not observed.

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LETTER TO THE EDITOR

Spin–orbit coupling and weak localisation in the 2D inversion layer of indium phosphide

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Abstract. We report measurements of the magnetoresistance (MR) of the 2D inversion layer of an InP MOSFET in the temperature range $T = 4.2$ to 0.5 K. For $k_F l \gtrsim 5$ we observe a positive MR at low magnetic fields $B < 0.015$ T and negative MR at higher values of B. We attribute this behaviour to the presence of strong spin–orbit coupling which, at $B = 0$, reduces the magnitude of the weak localisation. As B is increased, the spin–orbit interaction is progressively quenched, leading to a positive MR which eventually turns over into negative MR as the time reversal symmetry of the quantum interference (weak localisation) is destroyed. Our results can be qualitatively explained but precise agreement could not be obtained in the regime where the spin–orbit effect was dominant.

Analysis of the negative magnetoresistance has allowed us to observe the role of electron–electron scattering which could be expressed as two terms varying with temperature as T and T^2. The results do not support a suggestion that the T term can be expressed as $T\ln(T_1/T)$ where T_1 is a constant.

In two dimensions (2D), constructive electron quantum interference between multiple scattering events leads to a decrease of the 'metallic' conductance as a power of temperature T (Davies and Pepper 1982). For small changes this is the well known logarithmic correction initially predicted by Abrahams *et al* (1979) (for a review see Uren *et al* 1981a). For a square sample of side L, at $T = 0$ K, the 'normal' conductance $\sigma_0 = ne^2\tau/m^*$ becomes $\sigma = \sigma_0 + \Delta\sigma_{loc}$ where

$$\Delta\sigma_{loc} = -\frac{e^2\alpha}{\pi^2\hbar}\ln\frac{L}{l} \tag{1}$$

where n is the number of electrons per unit area, τ and l are the elastic scattering time and length respectively, m^* is the effective mass of the electron, and α is a constant of the order of 1.

At finite temperatures L is the inelastic scattering length L_{in}; in the temperature range of interest this is due to electron–electron scattering. If τ_{in} varies as T^{-p} then equation (1) is experimentally observable as

$$\Delta\sigma_{loc} = -\frac{e^2\alpha p}{2\pi^2\hbar}\ln\frac{T}{T_0} \tag{2}$$

where T_0 is an appropriate constant and p is a constant. For Landau–Baber electron–electron scattering the mean free time t_{in} varies as T^{-2}, hence L_{in} varies as T^{-1}. Increasing the disorder reduces p (Uren *et al* 1981a, b).

Electron–electron interactions in the presence of weak impurity scattering (Altshuler *et al* 1980a, b) cause the density of states $N(E)$ at the Fermi energy E_F to decrease logarithmically with decreasing temperature. The conductance correction $\Delta\sigma_{int}$ due to interactions is given by

$$\Delta\sigma_{int} = \frac{e^2}{4\pi^2\hbar}(2-2F)\ln\frac{T}{T'} \tag{3}$$

where T' is a temperature constant and F is the 2D electron–electron screening factor given by

$$F = \int_0^{2\pi} \frac{d\theta}{2\pi[1+(2k_F/K)\sin(\theta/2)]} \tag{4}$$

Here k_F is the Fermi wavevector and K, the inverse 2D screening length, is given by

$$K = \frac{e^2}{2\epsilon_0\epsilon_r}N(E) \tag{5}$$

where ϵ_r is the dielectric constant. The value of K is [illegible] in silicon inversion layers and F is near unity, resulting in a small interaction contribution in the absence of a magnetic field (Uren *et al* 1980, 1981a, b, Davies *et al* 1981). On the other hand III–V semiconductors have a smaller value of $N(E)$ due to a lighter mass and absence of valley degeneracy. This results in a smaller value of F and a significant interaction contribution. We have previously reported the coexistence of interaction and localisation effects in the 2D electron gas of a GaAs–GaAlAs heterojunction (Poole *et al* 1981). In this work we have used InP MOS devices ($\epsilon_r = 14$ for InP), the distinction of which is that the low mass ($m^* = 0.07\, m_0$) and absence of a valley degeneracy results in a value of K^{-1} of [illegible] 48 Å. This gives values of F between about 0.2 and 0.4 for the range of k_F investigated in this work. From equation (3) it is clear that, unlike Si, the contribution $\Delta\sigma_{int}$ will be significant in InP as it is in GaAs.

Application of a transverse magnetic field B produces negative magnetoresistance (MR) as the length scale is now determined by both L_{in} and the cyclotron length $L_c = \sqrt{(\hbar/eB)}$. (The contribution $\Delta\sigma_{int}$ is not significantly affected by an increase in magnetic field B so long as $g\beta B < kT$ (Davies *et al* 1981, Lee and Ramakrishnan 1982), which is the case in this work.) The applied B removes the time reversal symmetry of the backscattered and incident partial waves, thus breaking the coherence of the quantum interference effect and reducing the term $\Delta\sigma_{loc}$. In the presence of spin–orbit coupling a further conductance correction appears (Hikami 1980, Fukuyama 1981). This effect is predicted to result in an increase in conductance with decreasing temperature. However, as B is increased the spin–orbit coupling is quenched, leading to an increase in $\Delta\sigma$ [illegible], but at the same time (arising from the elimination of the contribution with opposite sign) the applied B will itself start breaking the time reversal symmetry, thus reducing $\Delta\sigma$ [illegible]. The relative importance of these two effects is described via the relevant scattering times [illegible] for elastic scattering of electrons off spin–orbit scattering sites and τ_{in}. If τ_{so} [illegible] it is possible to observe positive MR as B is increased from zero, which then turns over into [illegible] at a critical B where the coherent quantum interference is at a maximum. If [illegible] only negative MR is observed. If $\tau_{so} \ll \tau_{in}$ only positive MR is observed. [illegible] has been observed in thin films of Mg covered with Au

Table 1. Some parameters of the InP MOSFET measured at 4.2 K.

Gate bias V_G (V)	Fermi energy E_F (meV)	Carrier concentration n (cm^{-2})	Sheet resistance R_s ($\Omega/\square$)	Mobility μ ($cm^2 V^{-1} s^{-1}$)	Elastic scattering length l [illegible]	Dimensionless constant $k_F l$	Screening factor F
4	83	2.7×10^{12}	13970	[illegible]	25.7	1	0.354
8	173	5.6×10^{12}	4020	270	[illegible]	6	0.275
10	207	6.7×10^{12}	3355	[illegible]	121	7.9	[illegible]
15	305	9.9×10^{12}	[illegible]	[illegible]	[illegible]	7.9	[illegible]

(Bergmann 1982). Hikami *et al* (1980) and Maekawa and Fukuyama (1981) have theoretically considered the inclusion of spin–orbit scattering into the MR of a 2D system with weak localisation and obtained

$$\Delta\sigma_{\rm L} = (-e^2/2\pi^2\hbar)\{\psi(\tfrac{1}{2} + 1/\tau_1 a) - \psi(\tfrac{1}{2} + 1/\tau_2 a)$$

$$+ [\psi(\tfrac{1}{2} + 1/\tau_3 a) - \psi(\tfrac{1}{2} + 1/\tau_2 a)]/2\} \tag{6}$$

where

$$a = 4DeB/\hbar$$

D is the electron diffusion constant and ψ is the digamma function. The times τ_1, τ_2, τ_3 are composed as follows:

$$1/\tau_1 = 2/\tau_{so}^x + 2/\tau_{so}^z + 2/\tau_s^x + 1/\tau_{in}$$

$$1/\tau_2 = 2/\tau_s^x + 4/\tau_s^z + 1/\tau_{in}$$

$$1/\tau_3 = 2/\tau_s^x + 4/\tau_{so}^x + 1/\tau_{in}$$

where τ_s is the paramagnetic impurity scattering time and the indices z and x correspond to motion in the plane and perpendicular to the plane of the 2D gas respectively.

The device used in these experiments was an InP MOSFET (metal-oxide–semiconductor field effect transistor). This has a structure similar to the type of Si MOSFETs used for previous work of this nature (Uren *et al* 1981a). A Van der Pauw geometry device was used with a Silox grown SiO_2 dielectric of 500 Å thickness and Zn-doped p-type substrate. Electrical characterisation was performed at $T = 4.2$ K using the Hall effect and resistance measurements allowing us to obtain the parameters given in table 1. The electron mobility μ can be seen to be particularly low, being a maximum at $V_g = 10$ V. The product $k_F l$ was found to become virtually constant for values of gate bias V_g above +15 V since μ (i.e. l) was falling at approximately the same rate as k_F was increasing. This type of behaviour is also found in Si MOSFETs which exhibit strong surface roughness scattering. High values of V_g (and therefore E_F) push the electrons closer to the InP/SiO_2 interface where they are increasingly scattered by the non-uniform surface potential. This effect limited our measurements to a maximum $k_F l$ of 8. The high degree of surface roughness is thought to be due to the thermocompression process for bonding the sample contacts to the header. In this process the InP is heated to ~ 500 K which can cause indium to diffuse to the SiO_2–InP interface where it forms into 'puddles' creating considerable roughening of the interface. The $T = 300$ K pre-bonding peak mobility for the sample was > 1500 cm^2 V^{-1} s^{-1} which fell to ~ 300 cm^2 V^{-1} s^{-1} after bonding.

Figure 1 shows the variation of $\Delta R/R$ with B at $T = 1.25$ K for various values of V_g (hence E_F) where $\Delta R = R(B) - R(0)$ and $\Delta R = -\Delta\sigma R^2$. For $E_F \gtrsim 100$ meV the onset of positive MR at low B changing over to negative MR can be seen. The disappearance of the positive MR at lower values of E_F can be qualitatively explained as follows. As a first approximation we assume that τ_{so} is constant, independent of T and E_F, being a function of the lattice and impurity ions. For a 2D electron gas in the presence of impurities or defect scattering the electron–electron scattering rate τ_{ee} is given by (Uren *et al* 1981a, b)

$$1/\tau_{ee} = AkT/E_F\tau + B(kT)^2/\hbar E_F. \tag{7}$$

Here τ is the elastic scattering time, and the first term arises from elastic scattering introducing an indeterminacy in the electron momentum and a consequently enhanced rate of inelastic scattering with a low change of momentum (this behaviour was first predicted by Schmid 1974). The second (T^2) term is the familiar Landau–Baber law

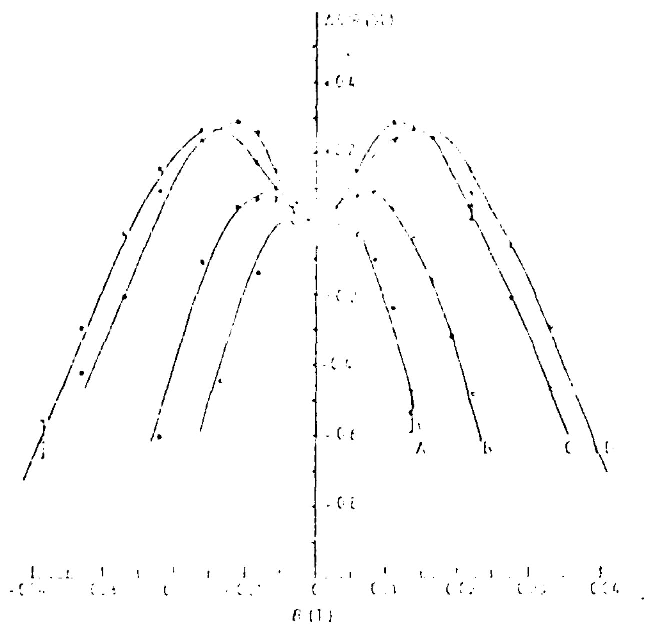

Figure 1. [illegible] at T [illegible] A, [illegible] B, [illegible] C, [illegible] D, [illegible]

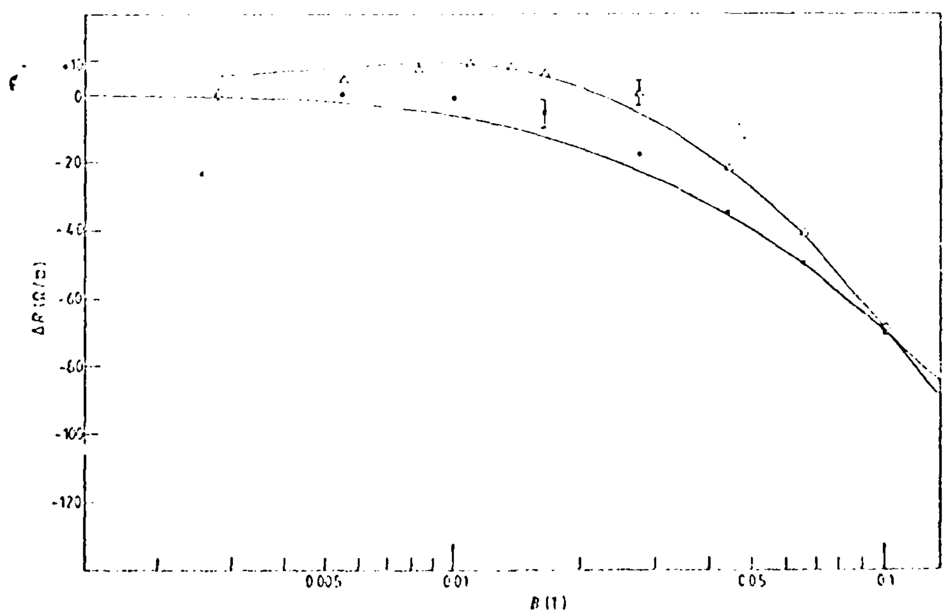

Figure 2. ΔR is plotted against B for $T = 4.2$ K (○) and $T = 1.25$ K (△). E_F [illegible] meV corresponding to $k_F l \approx 8$. *The full curves are theoretical calculations from equation (6) with* $\tau_o = 4.5 \times 10^{-12}$.

arising from scattering across the Fermi surface. Abrahams *et al* (1981) have also considered the effects of elastic scattering and find that $A \sim \ln T_1/T$ where T_1 is given by

$$T_1 = \frac{\pi^2}{8}\frac{m^*}{k^2 L_{\mathrm{F}}}\left(\frac{e^2}{\pi\epsilon_1\epsilon_0}\right)^2 (k_{\mathrm{F}}l)^3.$$

We assume that $\tau_{\mathrm{in}} \approx \tau_{\mathrm{e}}$, i.e. $\tau_{\mathrm{ee}} \ll \tau_{\mathrm{ep}}$ where τ_{ep} is the electron–phonon scattering (emission) time. This is clearly the case as found from the rate of electron heating by an applied electric field. From equation (6) we expect to see only negative MR when $\tau_{\mathrm{in}} <$ τ_{so}, which is most likely for low E_{F} as seen from equation (7). As E_{F} increases τ_{i} eventually becomes comparable to or greater than τ_{so} and we therefore start to observe the onset of positive MR.

In figure 2 we show results for the variation of ΔR with log B for various T at a fixed E_{F} of 308 meV. Figure 3 shows the equivalent results for $E_{\mathrm{F}} = 83$ meV. It can be seen that the higher E_{F} results show very little temperature dependence of $\Delta R/R$ and also a positive MR at low B and T. The solid lines are theoretical values obtained from equation (6) where we assume $\tau_{\mathrm{in}}^{-1} \sim T_{\mathrm{in}}$ and neglect the effect of magnetic scattering, i.e. $\tau_{\mathrm{s}}^{-1} \gg \tau_{\mathrm{so}}^{-1}, \tau_{\mathrm{i}}$. Although the agreement between the experimental points and theory is not good we can account for the differences between figures 2 and 3 as follows.

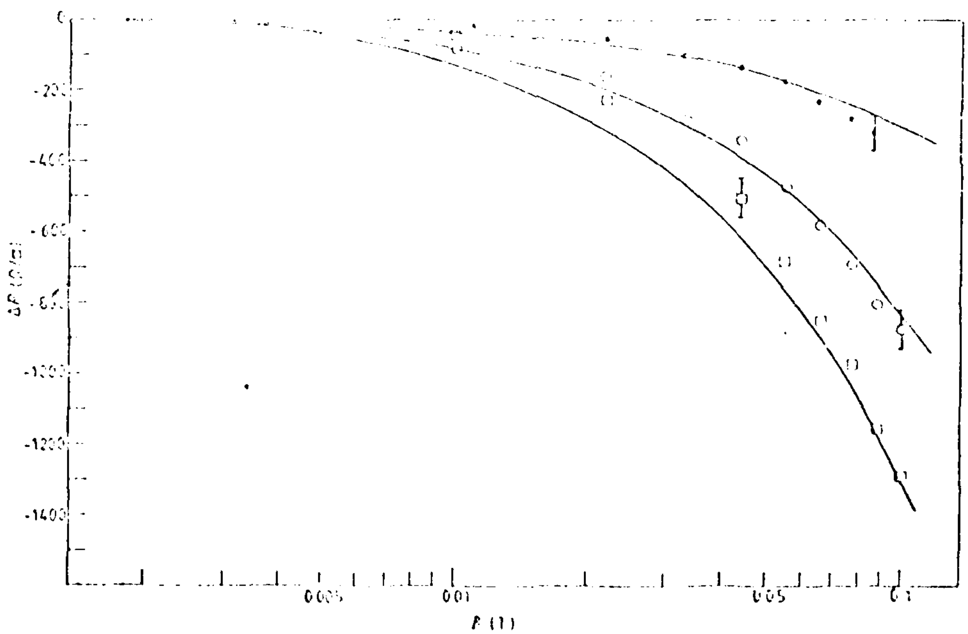

Figure 3. ΔR is plotted against B for $T = 4.2$ K (•), $T = 2.2$ K (○) and $T = 1.25$ K (□). $E_{\mathrm{F}} = 83$ meV corresponding to $k_{\mathrm{F}}l = 1$. The full curves are theoretical calculations from equation (6) with $\tau_{\mathrm{so}} = 8 \times 10^{-12}$ s.

First of all we consider the case of figure 3. Here τ_{in} is always less than τ_{so}; thus for all T we observe only negative MR which increases with falling T due to the increase in τ_{in} given by equation (6). τ_{so} has only a small effect on $\Delta R/R$. However, in figure 2 τ_{so} is

comparable with τ_φ at 4.2 K and becomes less than τ_φ at 1.25 K. Thus in this case as T is reduced the positive MR will increasingly dominate, causing the apparent lack of temperature dependence of $\Delta R/R$.

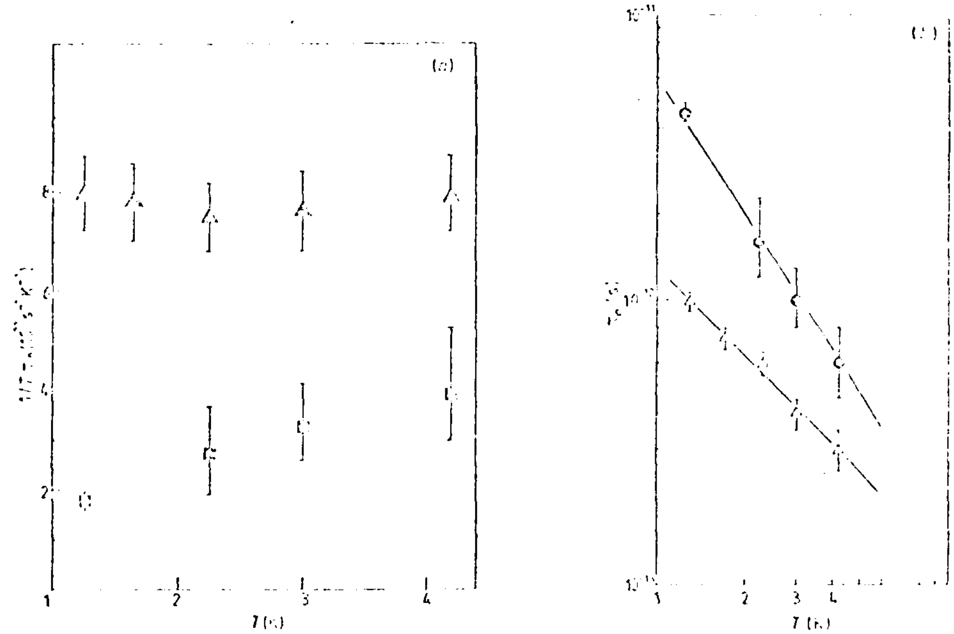

Figure 4. (a) The quantity $1/T\tau_{ee}$ is plotted against T for the case of E_1 = 83 meV (△) and E_1 = 305 meV (○). The values of τ_{ee} are obtained from the fits of equation (6) to the experimental results of figures 2 and 3. Straight lines through the points which do not pass through the origin indicate a $1/\tau_{ee} = A'T + B'T^2$ law. (b) ln τ_{ee} is plotted against ln T, allowing a value for p to be extracted. p = 1 and 1.4 for E_1 = 83 meV (△) and 305 meV (○) respectively.

In figure 4(*a*) we have plotted the quantity $1/T\tau_{ee}$ against T for E_1 = 83, 305 meV, where τ_{ee} has been extracted from the fits of equation (6) to the experimental data. This allows the values of the constants A' and B' to be found where equation (6) is rewritten as

$$1/\tau_{ee} = A'T + B'T^2.$$

From figure 4(*a*) the constants $A' = 8 \times 10^{11}\ s^{-1}\ K^{-1}$ and $B' = 0$ for E_1 = 83 meV ($k_1 l = 1$), $A' = 10^{11}\ s^{-1}\ K^{-1}$ and $B' = 0.72 \times 10^{11}\ s^{-1}\ K^{-2}$ for E_1 = 305 meV ($k_1 l = 8$). Thus in both cases the first term of equation (7) is dominating, indicating a system with a high degree of disorder. Plotting ln τ_φ against ln T will give a straight line relationship as shown in figure 4(*b*) over the small range of T we have considered. Expressing τ_φ as a composite of the two laws, the gradient of this plot gives a value for the constant p in equation (2) which becomes 1 and 1.4 for E_1 = 83 and 305 meV respectively. It is interesting to note that when $k_1 l \sim 1$ the T^2 component of electron–electron scattering is almost completely quenched. Presumably the fact that the indeterminacy Δk_1 is $\sim k_1$ results in the virtual elimination of collisions where the change in k_1 can be greater than

Δk_F. We note that the use of the expression derived by Abrahams *et al* for τ_{ε}^{-1} results in a calculated time which is approximately a factor of $T \ln T_1/T$ higher than the experimental value of 1 K. As the values of T_1 are ~2214 K ($k_F l \sim 1$) and 1.15×10^{5} K ($k_F l \sim 8$), agreement is poor. Furthermore the results for $k_F l \sim 1$ when plotted in the form of figure 4(*a*) should show that $1/\tau_{\varepsilon} T$ varies by a factor of 2 between 1 K and 4 K if the $\ln (T_1/T)$ term was present. However, the coefficients of the T component [illegible] vary by the expected ratio of 8 when $k_F l$ (or $E_F \tau$) is varied by a factor of 8. We thus conclude that these results do not support the existence of the $\ln T_1/T$ term but do support the coefficient being of the form $Ak/E_F \tau$ where A is a constant. Agreement between theory and experiment is obtained for $A \sim 1.5$.

The presence of strong spin–orbit scattering has an interesting effect upon the temperature dependence of the resistance. Hikami *et al* (1980) predicted that for $\tau_{so} \ll \tau_{\varepsilon}$, $\Delta\sigma_{so}$ varies at half the rate with T as does the original weak localisation $\Delta\sigma_{loc}$ (given by equation (2)) but with opposite sign. Thus the resistance will [illegible] decrease as $T \rightarrow 0$. To look for this effect we measured $\Delta R/R$ down to 0.5 K for a range of E_F, some results being shown in figure 5 normalised to 4.2 K. Clearly [illegible]

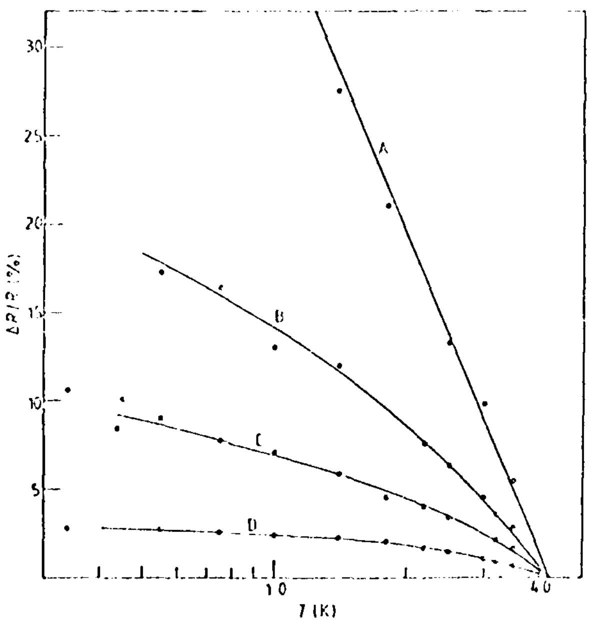

Figure 5. The percentage change in R, $\Delta R/R$, is plotted against T normalised to $T = 4.2$ K for various values of E_F. The full curves are for a guide only. A, $E_F = 83$ meV; B, 173 meV; C, 207 meV; D, 305 meV.

is a deviation from the logarithmic dependence upon T predicted by equation (2), especially at high E_F. The pronounced 'flattening' in $\Delta R/R$ with falling T is due to the increasing dominance of τ_{so} over τ_{ε}. No turnover in the direction of $\Delta R/R$ is seen. So far, however, we have neglected the contribution to $\Delta R/R$ from the electron–electron interaction given by equation (3). Even in the limit of strong spin–orbit

scattering no turnover in $\Delta R/R$ should be seen until its rate of change with T exceeds that from interactions. This condition will occur when, from equations (?) and (5),

$$\alpha p/2 > (1 - F) \tag{8}$$

where it is implicit that $\tau_{\varepsilon} \ll \tau_{\mathrm{so}}$ and $\Delta\sigma_{\mathrm{I}} \approx -\frac{1}{2}\Delta\sigma_{\mathrm{so}}$.

Unfortunately for our low E_{I} device $\Delta R/R$ should never change direction since, from table (1), equation (8) is not satisfied for any value of E_{I} (where $\alpha = 1$ and p varies from 1 to 1.4 for $E_{\mathrm{I}} = 83$ meV and 306 meV respectively. A high E_{I} InP structure should be able to show the reversal within the dilution refrigeration range of temperatures.

It is unknown at the present time whether the spin–orbit scattering in these devices originates from Zn ions or the host lattice. The spin–orbit scattering length $L = (D\tau_{\mathrm{so}})^{1/2} = 800-1600$ Å for the range of E_{I} measured. This is close to the mean separation of the Zn ions ~ 500 Å, making it possible that they are the source of the effect.

In conclusion, we have observed weak localisation and spin–orbit interactions in the 2D inversion layer of InP. Values for the characteristic times τ_{ε}, τ_{so} have been obtained although the theory appears inadequate when the spin–orbit effect is dominant.

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LETTER TO THE EDITOR

An experimental test of the scaling theory of conduction in two dimensions

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Abstract. The scaling theory of conduction has excited much interest in the problem of conduction in disordered systems. Basic to the theory is the assumption that a one-parameter scaling function exists. An experimental test of this is presented for two-dimensional transport in silicon inversion layers. The results are found to be inconsistent with such an assumption and we conclude that the function does not exist.

Abrahams *et al* (1979: AALR) have proposed a scaling theory of conduction. They defined the dimensionless conductance g, of a sample of size L and dimensionality d, by

$$\sigma = (e^2/\hbar) g L^{d-2}.$$

A scaling function β was then defined by

$$\beta = \frac{\partial \ln g}{\partial \ln L}.$$

The limiting forms of this function are known; AALR assumed that β was a smoothly varying function of g only between these limits. The assumption that the β function exists is basic to the scaling theory. This has not been rigorously proved theoretically, and computer studies have produced conflicting results, so we must turn to experiments to test the validity of this assumption.

The results of such a test are presented here. A 'scaling function' has been constructed from measurements of conductivity against temperature on a silicon inversion layer. In this system the electrons form a two-dimensional electron gas so that (as $d = 2$) $\sigma \propto g$ and analysis is simplified.

To perform the analysis some assumptions concerning the relationship between length and temperature must be made. As discussed by Anderson *et al* (1979) and Kaveh and Mott (1981a), the effective length is the distance the electron travels before being inelastically scattered. This gives, in the weak scattering limit, $L = (D\tau_i)^{1/2}$, where D is the diffusion coefficient and τ_i is the lifetime between inelastic scattering events. As $\tau_i \propto T^{-p}$ this gives $L \propto T^{-p/2}$. Magnetoresistance measurements (e.g. Kawaguchi and Kawaji 1980, Uren *et al* 1981) have shown that electron–electron scattering is the dom-

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inant inelastic mechanism, giving values of p between 1 and 2 depending on the value of $k_f l$.

The arguments presented here do not assume any particular form for this relationship, only that it is 'well behaved' in a sense which will be defined.

It is possible to define an 'experimental scaling function' by

$$\beta_e = \frac{\partial \ln g}{\partial \ln T}$$

$$g = h\sigma/e^2.$$

This can be related to the true β function by

$$\beta_e = \frac{\partial \ln g}{\partial \ln L} \frac{\partial \ln L}{\partial \ln T}$$

$$= \beta\gamma$$

where

$$\gamma = \frac{\partial \ln L}{\partial \ln T}.$$

With the conventional interpretation, $L = (D\tau_i)^{1/2}$, $\tau_i \propto T^{-p}$, $\gamma = p/2$, a constant. We make the more general assumption that γ is a function of g only ($\gamma = \gamma(g)$). Then, as by hypothesis $\beta = f(g)$, we find that β_e should also be a universal function, $\beta_e = \beta_e(g)$.

Such a function has been constructed from data previously published (Davies and Pepper 1982). As only a discrete number of data points were available the derivative was approximated to the gradient between adjacent points:

$$\beta_e = \frac{\Delta \ln g}{\Delta \ln T} = \frac{\Delta \ln \sigma}{\Delta \ln T}$$

and the g_e value was taken as the mean of the two values:

$$g_e = \frac{h}{e^2}\sigma_e.$$

Some representative results, for several carrier concentrations, are shown in figure 1. As can be seen, the results do not fit on a universal curve, rather the results for each carrier concentration fit on separate curves, with slopes opposite to that predicted by the scaling theory. It is also seen that the same value of β_e is found for different values of g. Here we stress that although there are errors in β_e the value of g is quite precise. Hence it is clear that there is not a well defined value of β_e for a particular value of g_e.

There are three possible conclusions from this, which will now be considered in turn.

(i) Electron–electron interactions may be significant. It is known (Altshuler *et al* 1980, Fukuyama 1980, Kaveh and Mott 1981b) that in the presence of impurity scattering electron–electron interactions also give a temperature dependence in the conductivity. The scaling theory is a single electron model and so does not include these effects. In the weak scattering limit it was first shown by Uren *et al* (1980) and also Uren *et al* (1981) that these effects are negligible in Si inversion layers in the absence of a magnetic field, with subsequent theoretical justification by Kawabata (1982). In the regime of strong localisation variable-range hopping has been widely observed (Mott *et al* 1975). This is also a single electron effect. As Coulomb effects are not important in these limiting cases

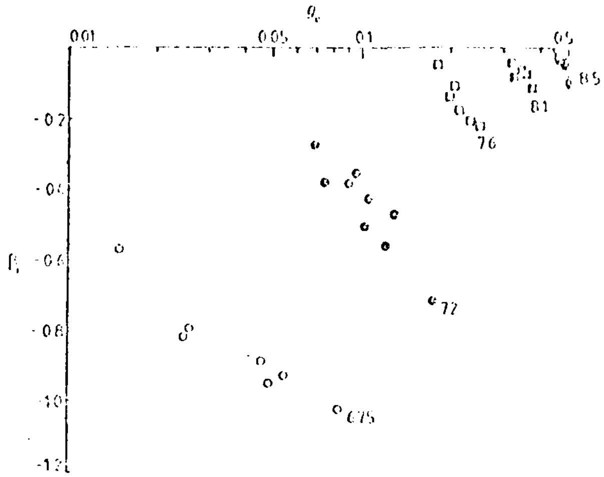

Figure 1. Here we show the experimental scaling function β_c as a function of the conductance g_c. The errors in β_c are larger than those in g_c, which are extremely small due to the precise measurement of the conductance. It is clearly seen that completely different values of β_c are found for the same g_c, indicating the absence of a universal function. The figures next to the points are values of n_s in units of 10^{15} m^{-2}. It is to be noted that at particular values of n_s, values of g_c are found below $0.1e^2/\hbar$, i.e. $2D\sigma_{min}$. However, this does not indicate strong localisation as at higher temperatures the value of g_c exceeds $0.1e^2/\hbar$, i.e. the Boltzmann conductance. The strong negative magnetoresistance found at low temperatures confirms that it is the quantum interference (weak localisation) causing the strong decrease in g_c and not strong (exponential) localisation.

it seems reasonable to assume that they are not important in the intermediate region either.

(ii) The assumption that $\gamma = \gamma(g)$ is invalid. As this is such an important assumption it is worth investigating further. In the weak scattering limit it is known that $L = (D\tau_i)^{1/2}$. There is, however, lack of agreement in the literature over what the appropriate form is when deviations from metallic conduction become important. If the unperturbed value of D is used, with $\tau_i \propto T^{-P}$, then $\gamma = P/2$. Alternatively the perturbed value of D can be used. As the conductivity is proportional to the diffusion coefficient, $D \propto g$, $L \propto g^{1/2}T^{-P/2}$, so that in this case $\gamma = P/(2-\beta)$. In both these cases $\gamma = \gamma(g)$, so the assumption is valid. It has been suggested by Abrahams *et al* (1981) that the electron–electron scattering rate $\tau_{e\text{-}e}$ in fact varies as

$$\frac{1}{\tau_{e\text{-}e}} \propto T \ln (T_1/T)$$

(with $T_1 = 3.6 \times 10^6 (k_F l)^3$ for the devices used). In this case

$$\gamma = \tfrac{1}{2}\left(1 + \frac{1}{\ln(T_1/T)}\right).$$

This quantity varies from a constant by less than 5% under all the experimental conditions; this is much less than the variation needed to explain the results. Furthermore, recent experiments do not support this formula and indicate that the $\ln T$ term is not

present or is considerably less than Abrahams *et al* (1981) propose (Poole *et al* 1982, Uren *et al* 1981, Davies and Pepper 1983).

Finally the question of the transition from a T^2 to a T electron–electron scattering rate must be considered (Uren *et al* 1981). In the high temperature and weak scattering limit a T^2 dependence is expected, with a T dependence in the opposite limit. This could produce deviations from a universal curve of the type found, but should become less important at small values of g, when the T dependence should always dominate. The opposite is in fact observed.

(iii) The third, and only remaining, possibility is that the assumption that a one-parameter scaling function exists is not valid.

It is interesting to consider what relationship between length and temperature would be required to make the results consistent with scaling. At low temperatures (~100 mK) a variation of L as $L \propto T^{-1/2}$ would be required, with at high temperatures (~1 K), a variation as $L \propto T^{-1}$. These powers must be the same for a wide range of conductivities and carrier densities. We are not aware of any predictions of such a temperature dependence.

We are thus forced to the conclusion that a single-parameter scaling theory is not applicable to these results.

In our earlier work (Davies and Pepper 1982) we found that the temperature dependence of the conductivity could be well described by a power law: $\sigma = \sigma_0(T/T_0)^\gamma$. This relationship should give a constant experimental scaling function: $\beta_c = \gamma$. However, the magnitude of β_c increases with g, which is inconsistent with this. Integration of $\beta_c \propto g$ gives the form

$$\sigma = \frac{\sigma_0}{1 - \gamma \ln(T/T_0)}$$

rather than $\sigma = \sigma_0(T/T_0)^\gamma$. This discrepancy prompted a careful check of the original curve fitting. Systematic deviations were found between the fitted curves and the data points. In all cases the line is above the points on intermediate temperatures and below at high and low temperatures. The form $\sigma = \sigma_0[1 - \gamma \ln(T/T_1)]$ was found to give deviations in the opposite directions.

The reason for this becomes apparent when the two forms are expanded as power series:

$$\sigma_0(T/T_0)^\gamma = \sigma_0(1 + \gamma \ln(T/T_0) + \tfrac{1}{2}[\gamma \ln(T/T_0)]^2 + \ldots)$$

$$\sigma_0/(1 - \gamma \ln(T/T_1)) = \sigma_0(1 + \gamma \ln(T/T_0) + [\gamma \ln(T/T_0)]^2 + \ldots).$$

The experimental results indicate that, as far as such an expansion is valid, the second-order term's prefactor has a value between $\frac{1}{2}$ and 1.

In conclusion, the validity of the assumption of the scaling theory of conduction has been experimentally tested. A 'scaling function' has been constructed but it is not a universal function, as the AALR theory requires, and overlapping values of β_c are found depending on g and Fermi energy (electron density). Various other possible causes for this discrepancy have been considered but none has been found capable of explaining it. We thus conclude that a one-parameter scaling function does not exist.

As previous work has indicated that exponential localisation is present in the band tail (Mott *et al* 1975), this present work indicates the presence of a 'mobility edge' separating expontentially localised states from weakly localised states. This has also been suggested on the basis of theory by Haydock (1981), Kaveh and Mott (1981a, b)

and Azbel (1982). It is also in agreement with our earlier results indicating excitation to a mobility edge, the location of which was not determined by an inelastic scattering cut-off, i.e. the location of the edge was not temperature-dependent. A quasi-mobility edge produced by inelastic scattering is expected if all states are exponentially localised, as suggested by the scaling theory.

We have enjoyed many stimulating discussions with Professor Sir Nevill Mott and Drs J H Davies and R V Haydock. This work was supported by SERC and in part by the European Research Office of the US Army. R A Davies acknowledges an SERC Research Studentship. M Kaveh thanks the Royal Society for a Guest Research Fellowship.

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LETTER TO THE EDITOR

Energy loss rate in silicon inversion layers

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Abstract. We report the results of measurements on the rate of heat loss from hot electrons in silicon inversion layers at low temperatures. The results are interpreted in terms of the generation of acoustic phonons and it is found that disorder has a significant effect on this mechanism. In the lowest disorder, high-temperature limit the energy relaxation time τ varies with electron temperature T_e as $\tau \propto T_e^{-3}$. In the highest disorder, lowest temperature limit τ varies as T_e^{-2}. The electron temperature is measured by the effect on the weak two-dimensional localisation which allows the experiments to be performed at low temperatures.

The silicon inversion layer has become very popular for investigating the properties of two-dimensional electron systems. It is particularly useful for experiments on the effect of disorder as the elastic mean free path may be varied simply by changing the carrier concentration. In this work we present experimental and theoretical results on the rate of energy loss from a hot two-dimensional electron gas in an inversion layer. The letter is divided into three main parts. The first explains the theory of the experiment; the second briefly describes the experimental techniques, and finally we propose an explanation of the observed energy loss rate.

According to the theories of localisation there is a correction to the conductivity in two dimensions (e.g. Gorkov *et al* 1979, Kaveh and Mott 1981):

$$\Delta\sigma = -(n_v \alpha e^2/2\pi^2\hbar)\ln(\tau_i/\tau) \qquad (1)$$

where τ_i is the inelastic scattering time, τ is the elastic (impurity) scattering time, n_v is the valley degeneracy and α is a constant of order $\frac{1}{2}$.

The inelastic scattering time follows a power law dependence on temperature, $\tau_i \propto T^{-p}$, giving the expression

$$\Delta\sigma = (n_v \alpha p e^2/2\pi^2\hbar)\ln(T/T_0). \qquad (2)$$

It has been shown by Altshuler *et al* (1980) and Fukuyama (1980) that in the presence of impurity scattering the Coulomb interaction produces a similar correction to the conductivity. These logarithmic terms were first observed by Dolan and Osheroff (1979) in thin metal films and it was later shown by Uren *et al* (1980) that both terms are present and can be separated by the effect of a magnetic field (Davies *et al* 1981, Uren *et al*

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1981b). For the present work it is not important which term dominates, only that the conductivity is determined by the electron temperature. However, we point out that in the absence of a magnetic field only localisation is important in silicon inversion layers (Uren *et al* 1981b).

There are three scattering mechanisms in operation in the experiment: impurity scattering which is almost temperature-independent; electron–electron scattering and inelastic electron–phonon scattering. The correction to the conductivity will depend on the fastest inelastic scattering rate. It has been shown by magnetoresistance measurements that the fastest inelastic scattering rate at low temperatures is electron–electron scattering (Kawaguchi and Kawaji 1980, Uren *et al* 1981a).

Under the effect of an electric field the electrons will be heated. The only mechanism that removes energy from the electron gas is inelastic electron–phonon scattering. The electron–electron scattering results in a redistribution of the energy and the establishment of an electron temperature. The electrons will be heated until the energy loss rate due to phonon emission balances the energy gained from the field. This effect was postulated by Anderson *et al* (1979) to explain the results of Dolan and Osheroff (1979). In this situation the electron temperature will rise until a steady state is established by the energy balance condition

$$\tau_e \sigma E^2 = \int_{T_1}^{T_e} c_e \, dT \tag{3}$$

where E is the electric field, T_1 and T_e are the lattice and electron temperatures respectively, c_e is the electron specific heat and τ_e is a characteristic energy loss time.

If the energy loss time follows a power law dependence on electron temperature of the form $\tau_e \propto T^{-p}$ then a logarithmic variation of conductivity with electric field is expected

$$\Delta\sigma = \frac{2 n_v \alpha p}{2+p} \frac{e^2}{2\pi^2 \hbar} \ln(E/E_0). \tag{4}$$

As the conductivity is determined solely by the electron temperature, comparison of the electric field and temperature dependences allows an electron temperature to be associated with each value of the electric field. Thus we see that the correction to the conductivity is being used as a thermometer to measure the electron temperature.

If we assume a free electron value for the specific heat:

$$c_e = 2\pi m k_B^2 T/3\hbar^2 = cT \tag{5}$$

then the expression can be evaluated as

$$\tau_e = \frac{R_\square c}{2E^2}(T_e^2 - T_1^2). \tag{6}$$

Hence τ_e as a function of electron temperature can be obtained from the experiment.

The measurements were performed on silicon MOSFETs using a dilution refrigerator to achieve the low temperatures used. Details of the method are given by Uren *et al* (1981b); the main points are summarised here.

The device used was a silicon gate MOSFET fabricated on the (100) surface. An implant of boron limited the peak mobility to 0.23 m^2 V^{-1} s^{-1}. The gate was a square of side 250 μm with two pairs of potential probes arranged at $\frac{1}{4}$ and $\frac{3}{4}$ of the length. These probes allowed four-terminal resistance measurements to be made; the results are all given as resistance per square.

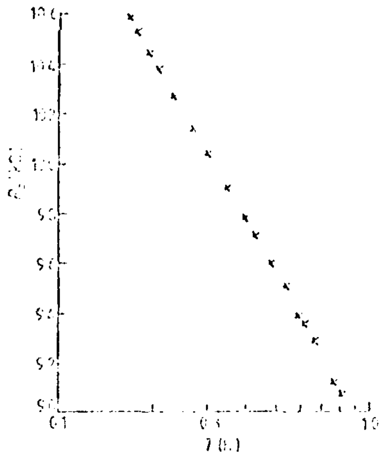

Figure 1. Variation of device resistivity with temperature plotted on a log scale, for carrier density $n_{2D} = 8.5 \times 10^{15}$ m^{-2}.

Resistances were measured by applying a constant AC current and measuring the voltage between the potential probes with phase sensitive detection. Voltages below 10 μV were required to give ohmic conduction at the lowest temperatures used. The range of temperature was 0.15 K–1.0 K and this was measured with a calibrated germanium thermometer. Figure 1 shows a typical temperature dependence: the logarithmic temperature dependence arising from the weak localisation is obvious.

With the device at the lowest temperature the variation of resistance with electric field was measured. This was performed by applying a DC bias current with the AC

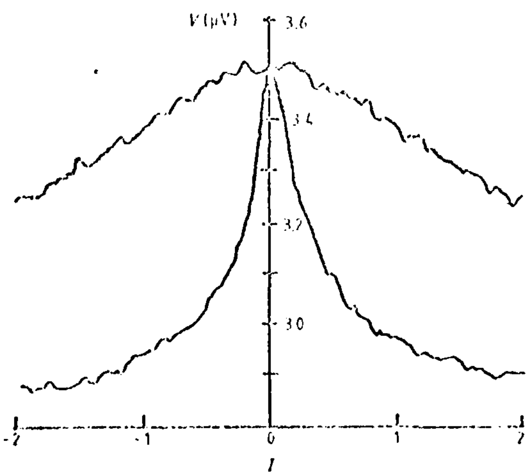

Figure 2. Differential resistance plotted against bias current, corresponding to figure 3. AC current = 10^{-9} A; the voltage should be multiplied by 3 to give resistivity. Units for the current are 10^{-7} A for the lower curve and 10^{-8} A for the upper curve. $n_{2D} = 8.5 \times 10^{15}$ m^{-2}. Lattice temperature, T_L = 170 mK.

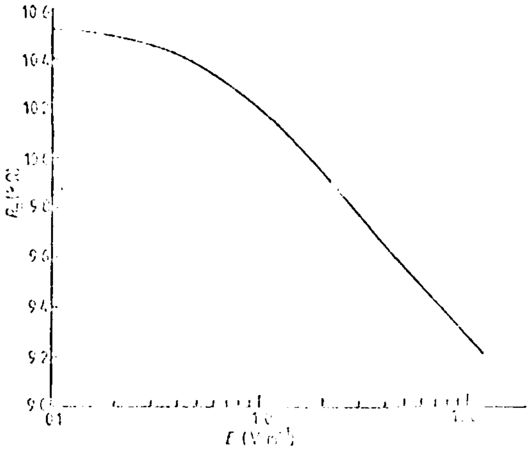

Figure 3. Variation of device resistivity with electric field. The curve is obtained by numerically integrating figure 2.

current. As the DC current was varied, the output from the phase sensitive detector was used to plot out the differential resistance $\partial V/\partial I(I)$. The plot of differential resistance is shown in figure 2. To obtain the resistance, rather than the differential resistance, this curve was numerically integrated. A problem which was encountered was that the DC voltage drop caused a change in the carrier density in the inversion layer. This was obviated by averaging over the two current directions. Figure 3 shows the variation of resistance with electric field; the logarithmic dependence is again found, as expected. This should be compared with the temperature dependence in figure 1.

A comment should be made here concerning the anomalous temperature dependence of electron–phonon scattering found in our earlier work (Uren *et al* 1980). Here $\tau_\varepsilon \propto T^{-4}$ was found. It now appears that this result is an artifact produced by the change

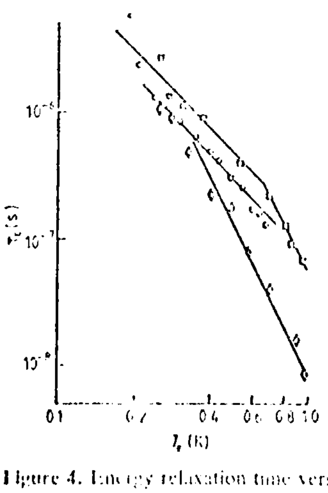

Figure 4. Energy relaxation time versus electron temperature for different values of the elastic mean free path l, on log scales. The lines drawn show $\tau_\varepsilon \propto T_e^{-3}$, $\tau_\varepsilon \propto T_e^{-2}$.

Table 1. Explanation of the symbols in figure 4.

	n_c ($10^{[illegible]}$ m^{-3})	$k_F l$	l ($10^{-[illegible]}$ m)
○	0.85	1.4	0.9
□	1.65	2.8	1.5
◇	1.70	6.5	2.8

k_F = Fermi wavevector and $k_F l$ is a measure of the disorder.

in carbon directly with electric fields; the current not being reversed in these earlier measurements.

For each point on the temperature dependence (figure 1), the electric field giving the same resistance can be found from the electric field dependence in figure 5. As the resistances are the same, the electron temperature must also be the same. This gives the electron temperature T_e corresponding to a particular value of E. The energy balance equation can then be used to obtain τ_e.

The results of this procedure, for three different carrier concentrations, are shown in figure 6 (see table 1). The temperature dependences can be approximately described by a power law $\tau_e \propto T^{-p}$ with p = 2, 3 and 4. This is in agreement with the results of other workers (Dynes [illegible]). Careful inspection shows that the points described as following a T^{-3} law actually follows a T^{-2} law at low temperature and T^{-4} at high temperature. We therefore take the interpretation that the energy loss time follows a T^{-4} dependence at high temperature and low disorder and a T^{-2} dependence at low temperature and high disorder with a transition in between. An explanation of this is presented below.

Several points should be made about the analysis.

(i) It is assumed that a well defined electron temperature exists. Other experiments (see for example Uren *et al* 1981c) have shown that the dominant inelastic scattering mechanism is electron–electron scattering, and that the lifetime for this is much shorter than the energy loss time found here, so the concept of electron temperature is valid.

(ii) The method is applicable irrespective of the source of the temperature dependence of resistance. The presence of weak localisation allows it to be extended to low temperatures where it normally cannot be applied.

(iii) It has been suggested by Kaveh *et al* (1981) and Tsuzuki (1981) that the electric field can influence weak localisation directly. This would invalidate the method. However, Altshuler *et al* (1981) have considered the same possibility and found that no such effect should exist. Experimentally Uren *et al* (1981b) and Bishop *et al* (1981) have found similar non-ohmic effects at large and small magnetic fields. The temperature dependence of resistance is due to weak localisation at small values of the magnetic field and due to interactions at larger values. The presence of similar non-ohmic effects when the temperature dependences are from different sources shows that heating is responsible for the effect. Even if a new length scale were established by the electric field in the absence of heating, it is much larger than the inelastic length and so does not play a role in this analysis.

To derive the energy relaxation time defined above we introduce the total energy loss rate $d\varepsilon/dt$.

$$d\varepsilon/dt = \tfrac{1}{2}c(T_e^2 - T_1^2)(1/\tau_1). \tag{7}$$

The value of $\mathrm{d}E/\mathrm{d}t$ is given formally by (Shinba et al 1982) by

$$\mathrm{d}E/\mathrm{d}t = \sum_{q} \hbar\omega(q)\,\dot{n} \tag{8}$$

where

$$\frac{\partial n(\omega)}{\partial t} = \frac{2\pi}{\hbar}\frac{4}{D(\omega)}\int \mathrm{d}^2k \sum_{q_z} |M(\omega, q_\parallel, q_z, k)|^2 \times \{[n(\omega)+1]f(k+q)[1-f(k)] - n(\omega)f(k)[1-f(k+q)]\} \times \delta(\varepsilon_{k+q} - \varepsilon_k - \hbar\omega_q). \tag{9}$$

k refers to electron states, ω, $q_\parallel$, q_z refer to phonon states, $q_\parallel$ being the phonon wavevector in the plane of the inversion layer and q_z the wavevector perpendicular to the layer. $f(k)$ is the Fermi distribution function of the electrons; n the Bose distribution function of the phonons. $M(\omega, q_\parallel, q_z, k)$ is the matrix element of the electron–phonon coupling. The sums over $q_\parallel$, q_z are for all phonons of energy ω and thus include the density of states factor.

It has been pointed out by Shinba et al that the major effect of the electron–electron scattering is to produce a well defined electron temperature T_e. The phonon distribution might also be expected to be hot since phonons are being generated by the hot electrons. However, most of these phonons are not propagating in the plane of the inversion layer, so the phonon distribution may be taken as characteristic of the lattice temperature T_l since the probability of these phonons being reabsorbed in the inversion layer is small.

At the low temperatures involved in the experiment only acoustic phonons of small q may be produced and they have energy $\omega \propto q$. It has been shown by Shinba et al (1982) that scatterings involving bulk phonons dominate the energy loss rate, even in the surface region, so we neglect the effects of surface phonons. We use a deformation potential for the electron–phonon coupling (see Ziman 1972):

$$|M_{k,k'}|^2 = K^2/\omega\,|V(K)|^2 \tag{10}$$

where $K = k' - k$ and $V(K)$ is the deformation potential which, for small K, is dependent only on the orientation of K and is independent of its magnitude. This needs to be modified to take into account the two-dimensional nature of the electron wavefunctions.

In the plane of the inversion layer, momentum is conserved, so $K \to q_\parallel$. In the direction perpendicular to the inversion layer momentum is not conserved. There is no coupling between different electron states due to q_z provided that $q_z \ll 1/w$, where w is the width of the inversion layer. The only effect of the finite width is to multiply the matrix element by a scaling factor given by the integral $\int_0^\infty \psi_z^* \exp(\mathrm{i}q_z z)\,\psi_z\,\mathrm{d}z$, where ψ_z is the wavefunction perpendicular to the inversion layer. For $q_z \ll 1/w$ this factor is a constant independent of q_z. The electrons scatter from a three-dimensional phonon distribution, the two-dimensional nature of the electron system being reflected in the effect on the matrix element of the electron–phonon coupling

$$|M(\omega, q)|^2 \Rightarrow |M(\omega, q_\parallel)|^2 \approx q_\parallel^2/\omega\,|V(q_\parallel)|^2. \tag{11}$$

When the elastic mean free path of the electron becomes shorter than the phonon wavelength, the matrix element falls (Pippard 1955). The correction factor multiplies the square of the matrix element by a factor ql. For the temperature range of our

experiment this factor must be included for all scatterings since $q_\parallel l$ is always $\gg 1$.

$$|F(\omega, q_z, q_\parallel, k)|^2 \rightarrow |I(\omega, q_\parallel)|^2 \rightarrow |M(\omega, q)|^2 q_\parallel l \tag{12}$$

$$|F(\omega, q_z, q_\parallel, k)|^2 \propto (q_\parallel^3 l/\omega)|V(q_\parallel)|^2. \tag{13}$$

We now proceed by converting equation (9) to an integration over energy. The phonon energies are much smaller than the Fermi energy, so scatterings take place between states with $|k'| = |k + q_\parallel|$. The electron mass is isotropic in the plane of the inversion layer and the electrons behave as a two dimensional free electron gas, the Fermi surface being a circle. Electrons in state k can scatter from phonons of energy $\hbar\omega$ to all states $k + q_\parallel$ as long as $q_\parallel = \hbar\omega/c$ where c is the velocity of sound, and as long as the energies of the states obey the energy conservation condition. Converting equation (9) to an energy interval gives

$$\frac{\partial n(\omega)}{\partial t} \propto \int \mathrm{d}\varepsilon f(\varepsilon + \hbar\omega)[1 - f(\varepsilon)]|F(\omega)|^2 \left\{\frac{\exp(\hbar\omega/kT_l) - \exp(\hbar\omega/kT_e)}{\exp(\hbar\omega/kT_l) - 1}\right\} \tag{14}$$

where

$$|F(\omega)|^2 \propto \int_0^{\hbar\omega/c} q_\parallel^3 \,\mathrm{d}q_\parallel \frac{l|V|^2}{\omega} = \left(\frac{\hbar}{c}\right)^4 \frac{l|V|^2}{4}\omega^3. \tag{15}$$

$|V|^2$ is the value of $|V(q_\parallel)|^2$ averaged over all orientations of $q_\parallel$. The integration over electron states round the Fermi circle is equivalent to averaging over all orientations of $q_\parallel$. It was shown earlier that $V(q_\parallel)$ is independent of the magnitude of $q_\parallel$, so $|V|^2$ is independent of the magnitude of $q_\parallel$.

The integration over energy in equation (14) can be performed, giving

$$\frac{\partial n(\omega)}{\partial t} \propto \frac{\hbar\omega}{[\exp(\hbar\omega/kT_e) - 1]}|F(\omega)|^2 \left\{\frac{\exp(\hbar\omega/kT_e) - \exp(\hbar\omega/kT_l)}{\exp(\hbar\omega/kT_l) - 1}\right\} \tag{16}$$

or

$$\frac{\partial n(\omega)}{\partial t} \propto |F(\omega)|^2 \left\{\frac{\hbar\omega}{\exp(\hbar\omega/kT_e) - 1} - \frac{\hbar\omega}{\exp(\hbar\omega/kT_l) - 1}\right\}. \tag{17}$$

As expected, if the temperatures of the electron and phonon gases are the same, $\partial n/\partial t$ is zero.

Finally, changing the sum over ω in $\mathrm{d}\varepsilon/\mathrm{d}t$ to an integral gives

$$\frac{\mathrm{d}\varepsilon}{\mathrm{d}t} \propto \int_0^\infty \mathrm{d}\omega \hbar\omega l|V|^2\omega^3 \left\{\frac{\hbar\omega}{\exp(\hbar\omega/kT_e) - 1} - \frac{\hbar\omega}{\exp(\hbar\omega/kT_l) - 1}\right\} \tag{18}$$

$$\frac{\mathrm{d}\varepsilon}{\mathrm{d}t} \propto \int_0^\infty \mathrm{d}\omega l|V|^2 \left\{\frac{\omega^5}{\exp(\hbar\omega/kT_e) - 1} - \frac{\omega^5}{\exp(\hbar\omega/kT_l) - 1}\right\} \tag{19}$$

$$\mathrm{d}\varepsilon/\mathrm{d}t = A(T_e^6 - T_l^6). \tag{20}$$

This gives

$$1/\tau_e = \frac{A(T_e^6 - T_l^6)}{\frac{1}{2}c(T_e^2 - T_l^2)} \tag{21}$$

or

$$\tau_e \propto T_e^{-4} \quad \text{for} \quad T_e \gg T_l. \tag{22}$$

Thus scattering between plane wave states gives the observed dependence of τ_ϵ on electron temperature for the sample with the longest elastic mean free path. The T_e^{-2} dependence of τ_ϵ is observed at low values of l (figure 4), which suggests that it is a result of the disorder. Kaveh and Mott (1981) give us a model for the electron states in a disordered two-dimensional system which provides a means of including disorder in calculations based on plane wave 'k' space; thus the model gives a simple means of extending 'k' space calculations to cover the disordered case. In the Kaveh and Mott theory an electron state $|k\}$ has the wavefunction

$$|k\} = \alpha\,|k\rangle + \sum_p \beta(p)\,l\,p\,|k+p\rangle \tag{23}$$

where $|\,\rangle$ represent plane wave states, p is a vector in the two-dimensional plane with $p_{min} = p = l^{-1}$ and $|\beta(p)|$ is independent of l and p.

If the electron states are not plane waves we cannot put $K = q$ in the deformation potential matrix element. We must evaluate the matrix elements using the actual eigenstates. If we use the Kaveh and Mott model for the eigenstates then the matrix element has the form

$$|M_{k'k}|^2 = 1/\alpha\,|\{k'|V|k\}|^2 \tag{24}$$

and the averaged value now is

$$\langle|M_{k'k}|^2\rangle = (|V|^2/\alpha)(q^2 + \mathrm{const}/l^2). \tag{25}$$

When we use this to derive $d\epsilon/dt$ we obtain

$$d\epsilon/dt = A(T_e^6 - T_l^6) + B/l^4(T_e^4 - T_l^4). \tag{26}$$

For small values of l the second term will dominate to given an energy relaxation time

$$1/\tau_\epsilon = \frac{B/l^4(T_e^4 - T_l^4)}{\frac{1}{2}\epsilon(T_e^2 - T_l^2)} \tag{27}$$

or

$$\tau_\epsilon \propto T_e^{-2} \quad \text{for} \quad T_e \gg T_l. \tag{28}$$

Further work is being carried out to determine the values of the coefficients A and B, which scale with l in the same way. It is believed that A, B scale as l^3, which would give

$$d\epsilon/dt = Cl^3(T_e^6 - T_l^6) + D/l(T_e^4 - T_l^4) \tag{29}$$

where C and D are constants.

This is consistent with the experimental results. In the low-disorder, high-temperature limit the energy loss rate increases with increasing l (corresponding to increasing carrier density). In the high-disorder, low-temperature limit the energy loss rate increases with decreasing l.

We have investigated the rate of loss of energy from a hot two-dimensional electron gas as a function of electron temperature for systems with varying degrees of disorder. It is found that the energy relaxation time τ_ϵ varies with electron temperature as T_e^{-4} in the low-disorder, high-temperature limit. This result is in agreement with the theoretical result for scattering between plane wave states via acoustic phonons. In the

high-disorder, low-temperature limit τ_ε is found to vary as T_c^{-2}, which is in agreement with a calculation using the Kaveh and Mott model for the electron states in a disordered two-dimensional system.

We thank Professor Sir Nevill Mott, Dr J H Davies and Dr M J Kelly for useful discussions. M C Payne and R A Davies acknowledge SERC Research Studentships. Low-temperature measurements were performed at the SERC Rutherford Laboratory; we thank Dr S F J Read and Mr G Regan for their help and advice. This work was supported by the SERC and in part by the European Research Office of the US Army.

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LOSS OF DIMENSIONALITY, LOCALISATION AND CONDUCTANCE OSCILLATIONS IN N-TYPE GaAs FET's

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We present new results for the transition from 3 dimensional (3D) conduction to 2D conduction in a GaAs FET. By applying a magnetic field, B, it is possible to observe 2 metal insulator transitions at low temperatures by (a) suppression of weak localisation at low B returning the system to metallic conduction and (b) shrinking of the donor wavefunctions at high B localising states at the Fermi energy. Magnetoresistance has been measured over 4 decades of B and for temperatures between 4.2 and 1.2 K, the results being in satisfactory agreement with current theories of localisation in 2D and 3D. We also present new conclusions for the anomalous oscillations in conductance with applied [illegible] bias observed in most GaAs FETs at low temperatures. The strength of the oscillations is related to the quality of the FET, being predominant in commercial microwave GaAs FETs.

If a voltage V_g, V_{sub}, is applied to the Schottky gate or substrate respectively of a GaAs FET the width of the conducting channel is reduced. Recently evidence has been presented that the narrowing results in a continuous transition from 3D to quasi 2D conduction with a progressive fall in Fermi energy, E_F (Poole et al., [1]). The results of the previous work were for GaAs FETs of doping density N_D [illegible] 10^{17} cm^{-3} and 5 [illegible] $10^{[illegible]}$ [illegible]. We [illegible] present results for N_D = 5.5 × $10^{[illegible]}$ cm^{-3} where, as before, E_F has been [illegible] the Shubnikov de Haas effect. In Fig. 1 the [illegible] energy difference ΔE [illegible] $E_n - E_o$ is plotted against channel thickness t, where E_n is the energy of the nth subband and E_o is the ground state subband energy. The theoretical fall in E_F as t → 0 is shown along with the experimental points which can be seen to be in satisfactory agreement. The value of E_F for a 3D free electron gas is also included for comparison. The subband energies and E_F were calculated using the semi classical WKB method which as Bergren found in our previous work is a good approximation to the more precise variational type calculation.

We may now proceed to compare the experimental magnetoresistance and temperature dependence of resistance of the GaAs FET with current theories of localisation in both 2D and 3D. Since our GaAs FET is doped just above the critical doping concentration for the Mott metal insulator transition it is possible to observe 3 distinct regions in the magnetoresistance at modest magnetic fields (< 15 T). Fig. 2 shows the log resistance plotted against B for various values of t, the 3 regions are indicated as I, II, III. Region I is the negative magnetoresistance which results from the breaking of the coherent interference of multiply backscattered electrons off impurity sites (weak localisation). This is most important in the 2D regime hence the increase in negative magnetoresistance as t decreases. It can be considered as an insulator-metal transition since weak localisation will destroy metallic conduction as [illegible] of decreasing temperature, the conductivity becoming zero at T = 0. The magnetic field suppresses weak localisation and returns the system to true metallic conduction being finite at T = 0, assuming interaction effects are only a weak perturbation. Region II consists of the Shubnikov de Haas oscillations observed in the 3D and 2D metallic system from which our data for Fig. 1 was extracted. Oscillations should appear when $\omega_c\tau \sim 1$, where ω_c is the cyclotron frequency and τ the elastic scattering time. In region III the donor wavefunction shrinks due to the high field by an amount sufficient to cause strong localisation at the Fermi energy and the system passes thro[illegible]

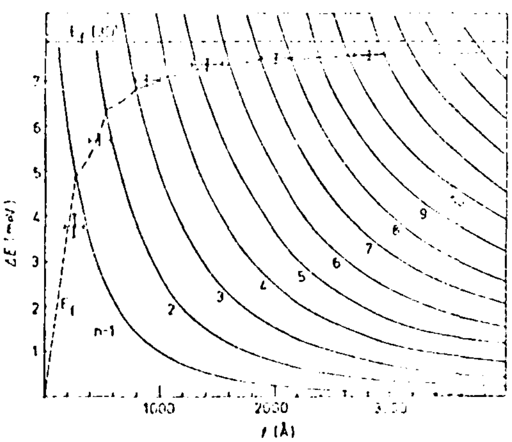

Fig. 1: Experimental and theoretical variation of E_F with t for N_D = 5.5 × 10^{16} cm^{-3} (T = 1.2K). The subband energies ΔE_n for n = 1,2,3 ... are referenced to the groundstate subband energy at n = 0.

another metal insulator transition. Measurement of the temperature dependence of region III confirms the previous results of Pepper [2] that the behaviour is well described by

$$R = R_0 \exp\left(\frac{\epsilon_0 + CB}{kT}\right) \qquad (1)$$

where C is a constant dependent on t in 3D and constant in 2D, ϵ_0 an energy and R_0 is the resistance at which the transition to activated behaviour occurs. This is the maximum metallic resistivity, resistance in 2D and 3D respectively. Regions I, II and III have also been observed for N_D up to 5.10^{17} cm^{-3} however magnetic fields $\gtrsim$ 25 T are needed to see region III in the 3D regime (Poole et al. [3]).

In Fig. 3 we show the negative magnetoresistance data of Fig. 2, region I, plotted as ln − ΔR/R against lnB (perpendicular $B_\perp$ and parallel $B_\parallel$ to the channel) for the case of t = 350Å (quasi 2D) and 2000Å (3D). The quantity ΔR is R(B) − R(o). It can be seen that in both 2D and 3D $-\Delta R/R \propto B_\perp^2$ for $B_\perp$ < 0.15 T. This turns over into a $\sqrt{B}$ dependence in 3D at higher B and into a log B dependence in 2D. The 2D negative magnetoresistance is always several times larger than the 3D. The behaviour for $B_\perp$ is consistent with the predictions of Hikami et al., [4] and Kawabata, [5] . In the 3D case Kawabata showed that coherent interference due to impurity scattering should lead to a negative magnetoresistance but much weaker than that in 2D giving

$$-\frac{\Delta R}{R} \propto R\sqrt{\delta} \qquad (2)$$

where the quantity $\delta = \frac{3\hbar c}{4 e l L_{in}^2 B} \ll 1$ (i.e. at high B, low T), and

$$-\frac{\Delta R}{R} \propto \left(\frac{\tau_{in}}{\tau}\right)^{3/2} \left(\frac{el}{\hbar}\right)^2 B^2 \qquad (3)$$

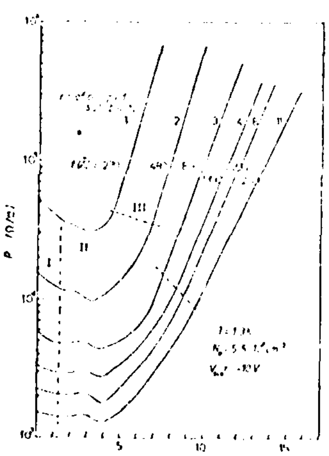

Fig. 2:
Plot of log R against $B_\perp$ as a function of t showing 3 distinct regions. I negative magnetoresistance, II Shubnikov de Haas oscillations, III activated, exponential behaviour.

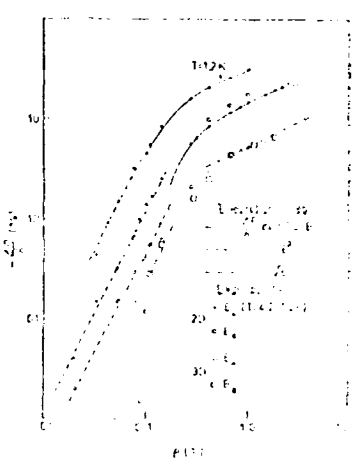

Fig. 3: [illegible] ΔR/R is plotted [illegible] $B_\perp$, $B_\parallel$ for t = 350Å [illegible] 2000Å [illegible] in one case T [illegible] 1.2 K. [illegible] dotted lines [illegible] a guide only.

when δ >> 1 (i.e. at low B, high T) where τ_{in}, τ is the inelastic scattering time and [illegible] respectively, m* the effective mass for [illegible] and l is the elastic scattering length. As T decreases then the inelastic diffusion length $L_{in} = \sqrt{Dt_{in}}$ will increase as T^{-p} where p [illegible] Landau-level scattering and is reduced by disorder (Poole et al., 1981). In the case of 2D Hikami showed that

$$-\frac{\Delta R}{R} = \frac{Re^2 \alpha}{2\pi^2 \hbar} \left[\ln \frac{\hbar}{4 \tau_{in} e D B} - \psi\left(\frac{1}{2} + \frac{\hbar}{4 \tau_{in} e D B}\right)\right] \quad (4)$$

where α is a constant (α $\lesssim$ 1), D is the 2D electron diffusion constant and ψ the digamma function. The solution of equation (4) gives a B^2 dependence at low B or high T and a log B dependence at high B or low T and is a good fit to our results. Figure 3 also includes ΔR/R for $B_\perp$ and T = 1.2 K in the 2D case to show that reducing T (i.e. increasing L_{in}) makes B more effective. This is true for $B_\parallel$, $B_\perp$ in both 3D and 2D. For $B_\parallel$ to the channel Fig. 3 shows that in 2D ΔR/R is small below B ~ 0.5 T and above this value rapidly increases, overtaking the $B_\perp$ result and then meeting it at B ~ 2.5 T. We might expect that there should be no difference in ΔR/R for $B_\perp$ or $B_\parallel$ in the 3D case and no negative magnetoresistance in the 2D case for $B_\parallel$. However, for weak localisation the important length parameter is the inelastic diffusion length L_{in}. As a rough criterion the temperature dependence of localisation is suppressed when the cyclotron length $L_c \gtrsim \sqrt{\hbar/eB}$ becomes less than L_{in}, this gives the field at which the negative magnetoresistance varies as log B. According to Al'tshuler and Aronov, [7] a 2D sample will show a log B behaviour in parallel field when $L_c \gtrsim \sqrt{t L_{in}}$ ($\gtrsim$0.1 to 0.2 T for our sample).

This effect arises from the blurring of the quantised energy levels. The data of Fig. 3 confirms this by showing that a higher $B_{\parallel}$ than $B_{\perp}$ is necessary to obtain the log B regime. It is clear that for the 3D case there is a difference between $B_{\perp}$ and $B_{\parallel}$ which we attribute to l_{in} being only a factor of ~6 less than t ($l_{in} \sim 350$Å, t = 2000Å). As T decreases we observe a larger difference as l_{in} becomes longer. Interaction effects in 2D and 3D in this system are considerably smaller than localisation and do not affect our conclusions. A more detailed analysis including such effects will be published later (Poole et al., [3]).

We now turn to the phenomenon of anomalous conductance oscillations in GaAs first discovered by Pepper, [8] in both GaAs FETs and later in Si inversion layers (Pepper et al., [9]; Pepper and Uren [10]). Oscillations in the channel conductance with applied gate bias are observed below T ≲ 10 K in almost all types of n-type GaAs FETs. Oscillations generally occur below a critical channel thickness and especially when conduction is activated, their amplitude increasing with decreasing temperature. We have now measured this effect in a wide range of both commercial and specially made GaAs FETs with substrate contacts (e.g. as used for the other results in this paper). The following new conclusions have been made.
(a) All of the commercial GaAs FETs tested showed oscillations. These included the following manufacturers: Hewlett Packard, NEC, Mitsubishi, Plessey, Avantek (all of these FETs have gate lengths L ≲ 4 μm). (b) Multiple gated FETs showed weaker oscillations than single gated. (c) Large area gates gave only weak oscillations e.g. we have investigated GaAs chips with one device of L = 200 μm and width W = 200 μm and one device of L = 12 μm and W = 200 μm the latter showing strongest oscillations. (d) The oscillations are always periodic in electron concentration $N_e = N_D t$ or electron separation r_{ee}, more usually the former. (e) Oscillations are observed for FETs over all of the investigated range for N_D ~ 5.10^{16} - 5.10^{17} cm^{-3}. No apparent relationship between periodicity and N_D has been found so far. (f) Application of B up to 10 T modifies the strength of the oscillations being completely reproducible for a fixed B. The periodicity remains essentially intact. (g) The strongest peaks are completely reproducible with temperature cycling. (h) Oscillations are suppressed by a high frequency a.c. electric field ≳ 500 MHz typically. We believe that these conclusions point to the effect being dominant in only very uniform, high quality (i.e. lack of defects etc.) material a necessary requirement being a uniform channel thickness over the device length. We are still uncertain as to the cause of the oscillations although electron ordering does not seem unrealistic. Figure 4 shows typical oscillations observed for a NEC FET and a linear plot of N_e against peak index N. The deviation from a straight line for high N_e is probably due to indiscernible 'missing' peaks.

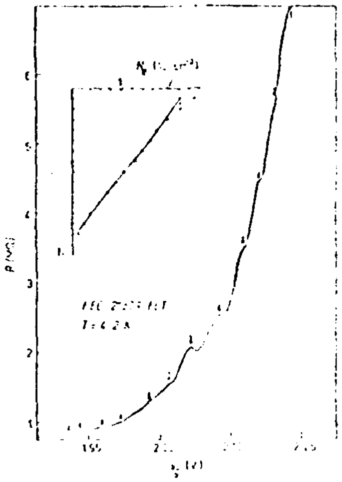

Fig. 4: Typical oscillations in resistance with gate bias for a commercial GaAs FET. The inset shows the resulting linear plot of peak index N against N_e.

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ELECTRON LOCALIZATION AND THE QUANTIZED HALL RESISTANCE IN SILICON INVERSION LAYERS

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We have investigated the formation of the plateau of quantized Hall resistance in the spin split minima and the lowest valley split minima of the ground Landau level of (100) Si inversion layers. The results in the spin gap are explained by a model based on Anderson localization in strong magnetic fields and on the existence of long range potential fluctuations. The behaviour of ρ_{xy} in the second, spin up, higher valley, level is discussed in relation to the compensation effect suggested in recent theories. Application of a field of 25 Tesla resulted in the delocalization of electrons in the lowest valley level and the appearance of the plateau of quantized Hall resistance in the lowest valley gap.

The observation of the quantized plateau behaviour of the Hall effect [1,2] in two-dimensional (2D) systems in strong magnetic fields has attracted much interest; in particular the possibility of obtaining a precise measurement of e^2/h. Theoretical discussions [3,4,5] suggest that the condition for the observation of the effect are localized states at the Fermi level, E_F, and extended states below which compensate the reduction of the Hall current caused by localized states. From an experimental point of view the plateau behaviour gives us important information on the electron localization in a 2D system in a strong magnetic field [6]. As a possible explanation for such electron localization there exist both experiments [7,8] and theories [9] which suggest the existence of a highly correlated state such as pinned Wigner solid (or glass) or pinned charge density wave. There also exist experimental [10-12] and theoretical [13] investigations which indicate the existence of a mobility edge in a strong magnetic field. We have investigated the relation between the formation of the plateau and the electron localization using n-type inversion layers formed on the (100) silicon surface. The samples used were MOS Hall devices having a 400 μm long and 50 μm wide channel and two pairs of Hall probes located equidistantly along the channel. Measurements were a.c. and were performed on the plateau appearing in the spin gap and in the lowest valley gap of the ground Landau level.

Figure 1 clearly illustrates the crucial role of extended states below E_F. When both the lowest spin, lower and higher valley, levels are localized, i.e. σ_{xx} throughout the levels decreases with decreasing temperature, the plateau is not formed. Here the conditions for 0.01 μA are such that the lowest two levels are localized. It is seen that increasing the electron temperature, by increasing the current, results in the appearance of the plateau in the spin gap indicating formation of extended states below E_F. For the experimental conditions the extended states are formed near the centre of the second valley level because the lowest valley level is invariably localized at magnetic fields up to 10 T. The role of increasing electron temperature has a number of possible effects such as screening, as discussed below, and thermal excitation.

Figure 2 shows the gate voltage dependence of the gradient $\Delta R_H/\Delta V_G$ in the spin gap. The position of the gap against the gate voltage was changed by both the application of the negative substrate bias and a change of magnetic field, B. The amount of the shift of the threshold voltage by the substrate bias was about half of the value expected. This arose from insufficient ionization of acceptors induced by substrate bias in the presence of LED illumination at these temperatures, but this does not affect

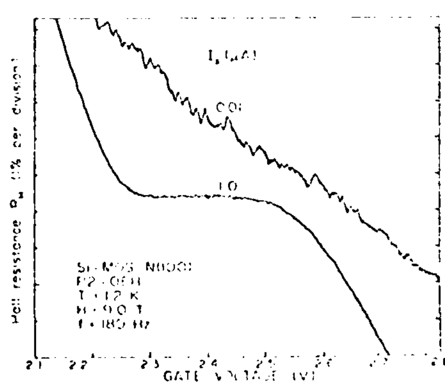

Fig. 1: The effect of increasing electron temperature on the plateau of the Hall resistance in the spin gap of the ground Landau level. The electrons were heated by increasing the current. For I_x = 0.01 μA there was still small metallic behaviour, when I_x was 1.0 μA the electron temperature was about 2 K.

our discussion. It is noted that both results show a [illegible] dependence on the gate voltage, which [illegible] the surface field is an important factor in plateau formation. Of course, increasing B increases the width of the Landau level, Γ, which according to the self consistent Born approximation [14] $\Gamma = (\hbar^2 \omega_c/2\pi\tau_1)^{1/2}$, ω_c is the cyclotron frequency and τ_1 is the elastic scattering time in zero magnetic field, and for the range of B shown Γ varies by only 20%.

Previous work on the non magnetic localization of carriers in the [illegible] tail of the inversion layer [15] showed that the application of a substrate bias altered the location of the mobility edge. Alteration of the bias, and [illegible] field, [illegible] the distance of the carriers from the interface and [illegible] the range of the scattering potential [illegible] that [illegible] the interface and decreasing the range of the potential [illegible] in E_c [illegible] level [illegible] decreasing [illegible] sequence [illegible] range of the [illegible] tant role [illegible] E_c, which has been [illegible] [17].

The weak character of the localization [illegible] centre of the Landau level [illegible] creation of the plateau at temperatures [illegible] 2.4 K, and by the role of frequency. [illegible] have investigated the effects of [illegible] by increasing the frequency of the current through the specimen. Figure 3 shows how a plateau is present at a frequency of 7 Hz but disappears at 500 Hz. This result implies that electrons

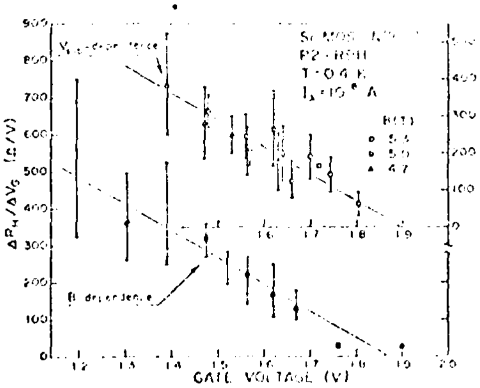

Fig. 2: Gate voltage dependence of the gradient $\Delta R_H/\Delta V_G$ in the spin gap region obtained by changing the magnetic field in the range of 4.0 T to 6.5 T, and at substrate bias of 0, −0.7, −1.0, −1.5 and −2.0 V for each magnetic field. Two straight lines exhibiting similar gradients are found.

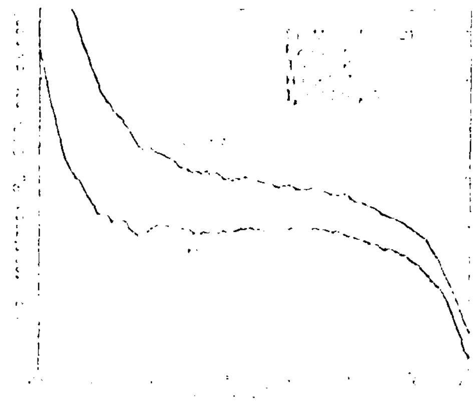

Fig. 3: [illegible]

[illegible] of the Landau level. [illegible] criterion for the existence of [illegible] $V_p < 2\Gamma_c$, where V_p is the [illegible] range fluctuation. [illegible] is fulfilled the locally extended states are connected together forming extended states spread throughout the sample, [illegible] both the short range and the long range potential fluctuations play important roles in 2D localization in strong magnetic fields.

The lowest spin, lowest valley level is invariably localized at magnetic fields up to 10 T. However a field of 25 T created a plateau in the valley gap at the expected value of [illegible], 813 [illegible] (h/e^2) as shown in Fig. 4. In a similar manner, the increase in magnetic field increased the peak value of σ_{xx} in the lowest level [18] from a low activated value to a metallic value of 1.8×10^{-6} mho/[illegible]. The existence of the plateau clearly indicates that extended states exist in the lowest valley level at least when E_F lies in the first valley gap. Although correlation effects may be important, the electronic state is not a pinned charge density wave [7,8] at

states are extended. The role of the magnetic field in delocalizing electrons in this level is consistent with an increase of the width of the extended state region resulting from the increased width of the Landau level. The long range potential fluctuations giving rise to the weak localization are, of course, unaffected by the magnetic field.

We have investigated the effect of the correction to the Hall current carried by extended states near the centre of the second level. This correction arises from the effects of the extended states compensating for the localized states in the tails. The carrier concentration n_{HALL} obtained from the Hall effect is plotted as a function of gate voltage in Fig. 5. It is seen that when the plateau is absent, due to all states being localized, n_{HALL} near the level centre varies with V_G in the manner expected from the capacitance relation indicating a "normal" Hall effect. However when states in the level centre are extended, and the plateau is found, this relation is not observed. The result suggests that the Hall effect is not compensating for the existence of the localized state in the tails. The origin of the observed "normal" Hall effect in localization regime, as in weak field case, is not yet clear. We note here that the plateau in the spin gap was observed when there was no plateau in the lowest valley gap. This implies that the extended states in the second level not only compensate for the localized states in this level, but also compensate for the lowest level which is entirely localized.

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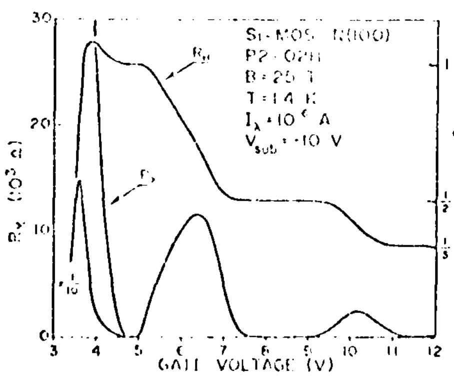

Fig. 4: Gate voltage dependence of the transverse resistance R_x and the Hall resistance R_H in the lowest three Landau levels at 25 Tesla. The principal plateau in the lowest valley gap is observed at V_G=4.8 V accompanying the gap region of zero resistance.

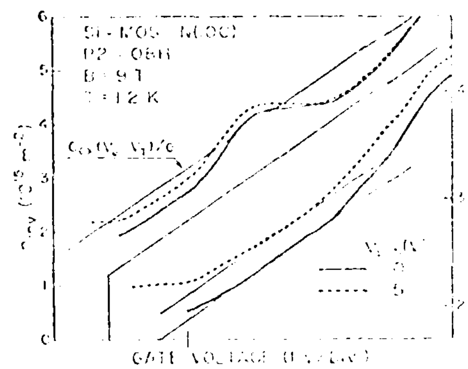

Fig. 5: Gate voltage dependence of inversion layer carrier density, n_{HALL} = B/eR_H, for two different substrate bias conditions, corresponding to the presence and absence of the plateau.

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This report describes work on two dimensional transport in Si and InP devices and spin dependent recombination in Si gate controlled p-n junctions. The two dimensional work covers investigations of the quantum Hall effect where we find that the effect is basically d.c. and can be removed by the application of a finite frequency. The cause of this effect is discussed in terms of a delocalization of electrons in the tails of Landau levels. The experiments indicate that localization in the tails of Landau levels...

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is caused by both disorder and the electron-electron interaction. Another aspect of the electron-electron interaction which has been investigated is the oscillatory conductance in inversion layers when charge is present at the Si-SiO_2 interface. It is suggested that Coulomb effects give rise to a contribution to the activation energy which oscillates with carrier concentration. Other topics in two dimensions which are discussed include the role of spin-orbit coupling in transport in the InP inversion layer and an investigation of the scaling theory of conduction. In this latter topic it is concluded that a one parameter scaling function does not exist. The existence of the weak 2D localization has also allowed an investigation of the rate of energy loss of hot electrons in Si inversion layers.

The other topic discussed is spin dependent recombination in Si gate controlled p-n junctions. The signal is found to be independent of frequency as suggested by theoretical models. We have also found a spin dependent generation signal of the same magnitude as the recombination signal. At present we do not have a theoretical model for this effect, future work will include both experiments and an attempt to produce a model accounting for both spin dependent recombination and generation.

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